Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.3-e-x-^m-a+b-sinh-c+d-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [102]. This is test number [161].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (102)	% 0.00 (0)
Mathematica	% 99.02 (101)	% 0.98 (1)
Maple	% 78.43 (80)	% 21.57 (22)
Maxima	% 82.35 (84)	% 17.65 (18)
Fricas	% 76.47 (78)	% 23.53 (24)
Sympy	% 30.39 (31)	% 69.61 (71)
Giac	% 52.94 (54)	% 47.06 (48)
Mupad	% 28.43 (29)	% 71.57 (73)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

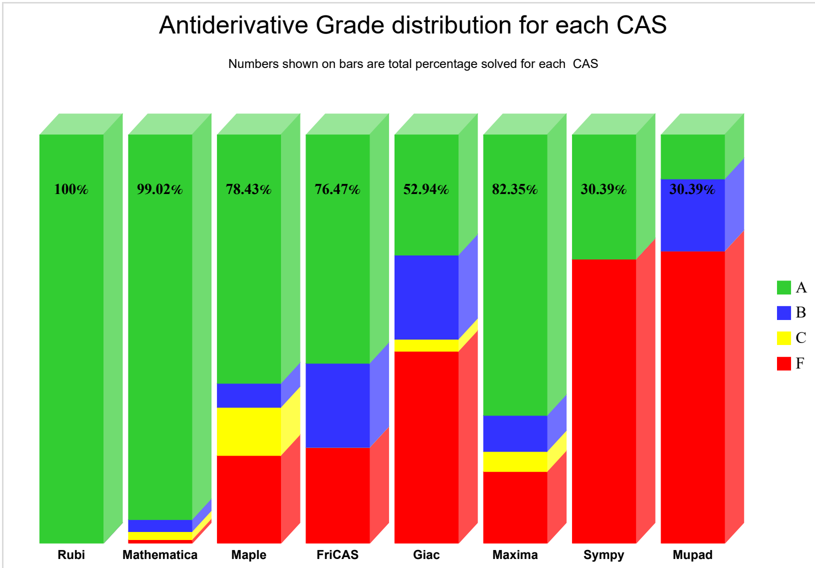
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

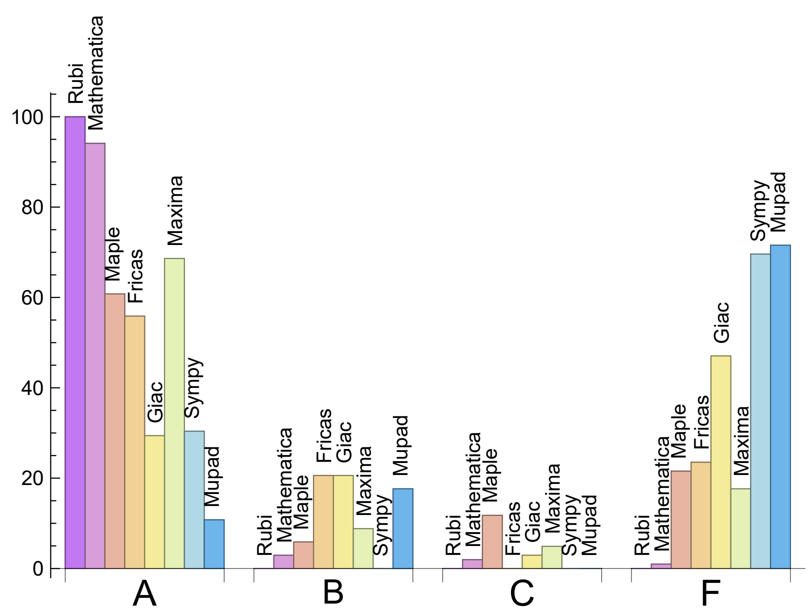
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	94.12	2.94	1.96	0.98
Maple	60.78	5.88	11.76	21.57
Maxima	68.63	8.82	4.90	17.65
Fricas	55.88	20.59	0.00	23.53
Sympy	30.39	0.00	0.00	69.61
Giac	29.41	20.59	2.94	47.06
Mupad	10.78	17.65	0.00	71.57

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	22	100.00 %	0.00 %	0.00 %
Maxima	18	100.00 %	0.00 %	0.00 %
Fricas	24	100.00 %	0.00 %	0.00 %
Sympy	71	95.77 %	4.23 %	0.00 %
Giac	48	100.00 %	0.00 %	0.00 %
Mupad	73	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

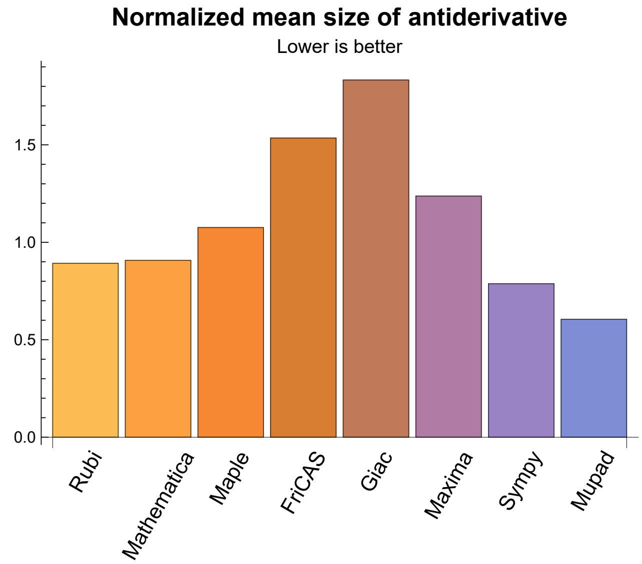
1.3 Performance

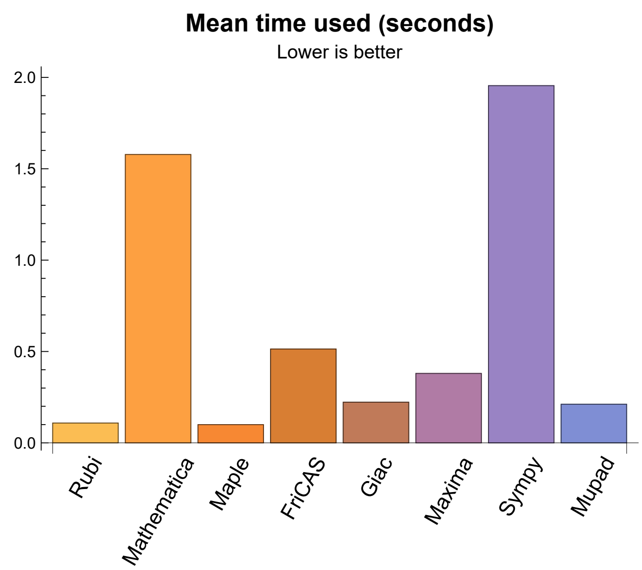
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	85.72	0.89	68.00	1.00
Mathematica	1.58	89.10	0.91	65.00	0.93
Maple	0.10	103.38	1.08	66.50	1.08
Maxima	0.38	95.08	1.24	58.50	0.90
Fricas	0.51	134.28	1.53	70.00	1.31
Sympy	1.95	42.13	0.79	22.00	1.00
Giac	0.22	161.67	1.83	57.00	1.44
Mupad	0.21	24.83	0.60	13.00	0.80

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

$\{23, 27, 40, 56, 74, 75, 77, 79, 83, 91, 92\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {78}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

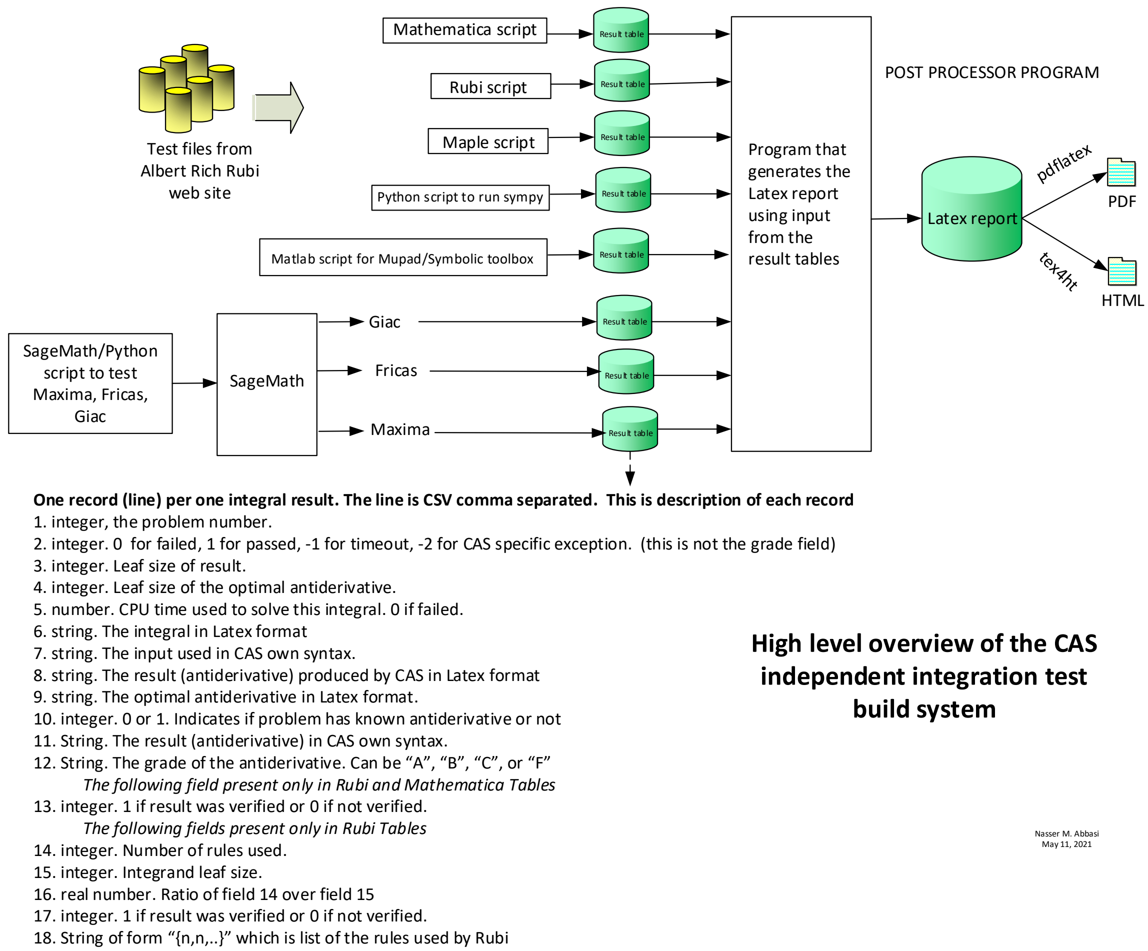
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

B grade: { 3, 24, 53 }

C grade: { 101, 102 }

F grade: { 37 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 61, 67, 72, 74, 75, 77, 79, 83, 84, 85, 86, 87, 91, 92, 95, 100 }

B grade: { 35, 36, 93, 94, 98, 99 }

C grade: { 26, 39, 55, 58, 59, 60, 62, 63, 82, 88, 89, 90 }

F grade: { 24, 25, 37, 38, 53, 54, 64, 65, 66, 68, 69, 70, 71, 73, 76, 78, 80, 81, 96, 97, 101, 102 }

2.1.4 Maxima

A grade: { 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 79, 83, 84, 85, 86, 87, 91, 92, 93, 94, 98, 99, 100 }

B grade: { 1, 2, 4, 17, 22, 88, 89, 90, 95 }

C grade: { 34, 35, 36, 50, 52 }

F grade: { 24, 25, 26, 37, 38, 39, 53, 54, 55, 76, 78, 80, 81, 82, 96, 97, 101, 102 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 42, 44, 46, 47, 48, 50, 52, 56, 57, 67, 72, 74, 75, 77, 79, 83, 87, 88, 90, 91, 92, 93, 94, 95, 98, 99, 100 }

B grade: { 2, 6, 9, 13, 16, 20, 22, 41, 43, 45, 49, 51, 61, 84, 85, 86, 89, 96, 97, 101, 102 }

C grade: { }

F grade: { 37, 38, 39, 53, 54, 55, 58, 59, 60, 62, 63, 64, 65, 66, 68, 69, 70, 71, 73, 76, 78, 80, 81, 82 }

2.1.6 Sympy

A grade: { 1, 3, 8, 10, 15, 17, 22, 23, 27, 28, 32, 33, 34, 35, 36, 40, 48, 50, 52, 56, 57, 74, 75, 77, 83, 91, 92, 93, 94, 95, 100 }

B grade: { }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 51, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 96, 97, 98, 99, 101, 102 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 9, 11, 12, 16, 17, 18, 19, 22, 23, 27, 28, 40, 50, 56, 57, 74, 75, 77, 79, 83, 87, 91, 92, 95, 100 }

B grade: { 7, 8, 10, 14, 15, 21, 29, 30, 31, 32, 33, 34, 35, 36, 42, 44, 48, 93, 94, 98, 99 }

C grade: { 88, 89, 90 }

F grade: { 6, 13, 20, 24, 25, 26, 37, 38, 39, 41, 43, 45, 46, 47, 49, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 96, 97, 101, 102 }

2.1.8 Mupad

A grade: { 23, 27, 40, 56, 74, 75, 77, 79, 83, 91, 92 }

B grade: { 1, 3, 8, 10, 15, 17, 22, 28, 33, 34, 35, 36, 48, 50, 52, 57, 95, 100 }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 24, 25, 26, 29, 30, 31, 32, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 51, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 93, 94, 96, 97, 98, 99, 101, 102 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	45	81	29	36	47	28
normalized size	1	1.00	0.91	1.32	2.38	0.85	1.06	1.38	0.82
time (sec)	N/A	0.036	0.034	0.017	0.307	0.504	0.804	0.258	0.088

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	74	110	190	0	75	-1
normalized size	1	1.00	0.97	1.07	1.59	2.75	0.00	1.09	-0.01
time (sec)	N/A	0.041	0.068	0.075	0.305	0.516	0.000	0.358	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	31	14	13	13	19	25	13
normalized size	1	1.00	2.07	0.93	0.87	0.87	1.27	1.67	0.87
time (sec)	N/A	0.016	0.010	0.004	0.300	0.722	0.214	0.389	0.374

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	86	48	0	41	-1
normalized size	1	1.00	0.85	0.75	1.62	0.91	0.00	0.77	-0.02
time (sec)	N/A	0.018	0.036	0.030	0.302	0.462	0.000	0.302	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	39	0	24	-1
normalized size	1	1.00	0.92	1.08	0.96	1.56	0.00	0.96	-0.04
time (sec)	N/A	0.035	0.014	0.020	0.373	0.484	0.000	0.311	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	70	54	184	0	0	-1
normalized size	1	1.00	1.06	1.06	0.82	2.79	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.062	0.032	0.319	0.480	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	58	39	71	0	109	-1
normalized size	1	1.00	0.90	1.38	0.93	1.69	0.00	2.60	-0.02
time (sec)	N/A	0.092	0.042	0.022	0.427	0.459	0.000	0.643	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	42	55	59	56	78	117	42
normalized size	1	1.00	0.82	1.08	1.16	1.10	1.53	2.29	0.82
time (sec)	N/A	0.050	0.104	0.055	0.329	0.480	1.550	0.545	0.103

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	90	95	427	0	97	-1
normalized size	1	1.00	1.02	0.91	0.96	4.31	0.00	0.98	-0.01
time (sec)	N/A	0.096	0.216	0.089	0.430	0.444	0.000	0.909	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	38	29	60	56	22
normalized size	1	1.00	0.87	1.10	1.23	0.94	1.94	1.81	0.71
time (sec)	N/A	0.027	0.023	0.013	0.333	0.522	0.438	0.791	0.379

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	86	51	56	73	0	58	-1
normalized size	1	1.00	1.10	0.65	0.72	0.94	0.00	0.74	-0.01
time (sec)	N/A	0.046	0.070	0.052	0.429	0.445	0.000	0.563	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	34	31	49	0	35	-1
normalized size	1	1.00	0.89	0.92	0.84	1.32	0.00	0.95	-0.03
time (sec)	N/A	0.059	0.020	0.055	0.412	0.412	0.000	0.344	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	94	86	61	396	0	0	-1
normalized size	1	1.00	1.07	0.98	0.69	4.50	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.223	0.069	0.381	0.500	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	69	36	90	0	126	-1
normalized size	1	1.00	0.81	1.21	0.63	1.58	0.00	2.21	-0.02
time (sec)	N/A	0.121	0.090	0.057	0.378	0.458	0.000	0.160	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	58	93	100	94	92	183	70
normalized size	1	1.00	0.73	1.18	1.27	1.19	1.16	2.32	0.89
time (sec)	N/A	0.083	0.127	0.091	0.353	0.464	2.772	0.209	0.126

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	184	157	162	904	0	166	-1
normalized size	1	1.00	1.15	0.98	1.01	5.65	0.00	1.04	-0.01
time (sec)	N/A	0.138	0.302	0.122	0.433	0.432	0.000	0.197	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	62	46	44	56	28
normalized size	1	1.00	1.00	0.85	1.88	1.39	1.33	1.70	0.85
time (sec)	N/A	0.033	0.013	0.015	0.324	0.519	0.783	0.238	0.060

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	136	86	91	112	0	95	-1
normalized size	1	1.00	1.09	0.69	0.73	0.90	0.00	0.76	-0.01
time (sec)	N/A	0.072	0.129	0.059	0.397	0.443	0.000	0.308	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	50	83	0	50	-1
normalized size	1	1.00	0.89	1.00	0.91	1.51	0.00	0.91	-0.02
time (sec)	N/A	0.094	0.034	0.077	0.426	0.439	0.000	0.244	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	204	149	102	892	0	0	-1
normalized size	1	1.00	1.50	1.10	0.75	6.56	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.315	0.097	0.442	0.507	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	120	58	160	0	223	-1
normalized size	1	1.00	0.99	1.32	0.64	1.76	0.00	2.45	-0.01
time (sec)	N/A	0.214	0.118	0.084	0.428	0.498	0.000	0.189	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	52	126	154	94	108	52
normalized size	1	1.00	1.00	0.78	1.88	2.30	1.40	1.61	0.78
time (sec)	N/A	0.047	0.025	0.119	0.424	0.415	7.686	0.230	0.474

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	2.506	0.052	0.000	0.431	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	735	0	0	252	0	0	-1
normalized size	1	1.00	3.43	0.00	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.201	12.547	0.239	0.000	0.594	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	152	0	0	174	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.579	0.163	0.000	0.753	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	98	77	0	124	0	0	-1
normalized size	1	1.00	1.03	0.81	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.153	0.100	0.000	0.452	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	2.965	0.071	0.000	0.499	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	19	25	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.27	1.67	0.87
time (sec)	N/A	0.020	0.008	0.005	0.301	0.420	0.771	0.216	0.382

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	130	47	93	0	534	-1
normalized size	1	1.00	0.90	1.67	0.60	1.19	0.00	6.85	-0.01
time (sec)	N/A	0.143	0.067	0.056	0.367	0.417	0.000	0.394	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	93	44	83	0	313	-1
normalized size	1	1.00	0.90	1.55	0.73	1.38	0.00	5.22	-0.02
time (sec)	N/A	0.107	0.046	0.048	0.349	0.419	0.000	0.218	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	56	36	58	0	173	-1
normalized size	1	1.00	1.00	1.70	1.09	1.76	0.00	5.24	-0.03
time (sec)	N/A	0.076	0.020	0.046	0.359	0.441	0.000	0.138	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	24	39	17	44	-1
normalized size	1	1.00	1.00	1.29	1.14	1.86	0.81	2.10	-0.05
time (sec)	N/A	0.032	0.010	0.043	0.394	0.396	1.029	0.408	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	27	13
normalized size	1	1.00	1.00	1.08	1.00	1.15	1.15	2.08	1.00
time (sec)	N/A	0.017	0.004	0.004	0.375	0.417	0.982	0.423	0.367

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	44	48	34	29	95	29
normalized size	1	1.00	1.00	1.52	1.66	1.17	1.00	3.28	1.00
time (sec)	N/A	0.030	0.028	0.019	0.407	0.510	1.769	0.241	0.380

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	94	47	43	46	214	67
normalized size	1	1.00	0.85	2.04	1.02	0.93	1.00	4.65	1.46
time (sec)	N/A	0.054	0.048	0.016	0.353	0.382	2.963	0.201	0.427

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	48	165	48	53	61	386	85
normalized size	1	1.00	0.77	2.66	0.77	0.85	0.98	6.23	1.37
time (sec)	N/A	0.080	0.065	0.017	0.388	0.385	4.899	0.188	0.442

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.251	180.001	0.200	0.000	0.473	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.256	0.184	0.000	0.425	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	70	0	0	0	0	-1
normalized size	1	1.00	0.94	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.087	0.109	0.000	0.422	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	3.418	0.086	0.000	0.449	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	138	62	323	0	0	-1
normalized size	1	1.00	0.98	1.33	0.60	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.112	0.079	1.339	0.435	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	93	44	89	0	353	-1
normalized size	1	1.00	0.90	1.50	0.71	1.44	0.00	5.69	-0.02
time (sec)	N/A	0.113	0.044	0.042	0.507	0.428	0.000	0.200	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	103	58	267	0	0	-1
normalized size	1	1.00	0.98	1.20	0.67	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.088	0.048	0.386	0.426	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	58	39	63	0	193	-1
normalized size	1	1.00	0.93	1.38	0.93	1.50	0.00	4.60	-0.02
time (sec)	N/A	0.082	0.026	0.025	0.390	0.470	0.000	0.184	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	70	70	71	228	0	0	-1
normalized size	1	1.00	1.04	1.04	1.06	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.067	0.048	0.360	0.560	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	24	39	0	0	-1
normalized size	1	1.00	1.00	1.08	0.96	1.56	0.00	0.00	-0.04
time (sec)	N/A	0.033	0.015	0.029	0.397	0.431	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	44	62	52	0	0	-1
normalized size	1	1.00	0.88	0.77	1.09	0.91	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.034	0.039	0.458	0.545	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	27	13
normalized size	1	1.00	1.00	0.93	0.87	1.13	1.47	1.80	0.87
time (sec)	N/A	0.019	0.005	0.005	0.320	0.470	2.738	0.165	0.375

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	82	62	251	0	0	-1
normalized size	1	1.00	0.99	1.09	0.83	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.073	0.050	0.388	0.512	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	55	48	40	37	43	58
normalized size	1	1.00	1.00	1.62	1.41	1.18	1.09	1.26	1.71
time (sec)	N/A	0.034	0.030	0.031	0.475	0.536	7.672	0.192	0.406

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	97	117	62	313	0	0	-1
normalized size	1	1.00	1.04	1.26	0.67	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.123	0.058	0.775	0.454	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	73	47	50	51	0	74
normalized size	1	1.00	0.94	1.55	1.00	1.06	1.09	0.00	1.57
time (sec)	N/A	0.059	0.050	0.048	0.375	0.472	19.216	0.000	0.436

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1039	0	0	0	0	0	-1
normalized size	1	1.00	5.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	24.294	0.197	0.000	0.537	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	122	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.829	0.183	0.000	0.557	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	77	0	0	0	0	-1
normalized size	1	1.00	0.97	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.144	0.103	0.000	0.572	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	3.397	0.079	0.000	0.515	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	11	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	1.38	0.75
time (sec)	N/A	0.012	0.003	0.009	0.442	0.561	0.263	0.172	0.400

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	77	73	0	0	0	-1
normalized size	1	1.00	1.17	1.03	0.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.098	0.126	0.563	0.523	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	69	73	0	0	0	-1
normalized size	1	1.00	1.17	0.92	0.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.085	0.089	0.508	0.414	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	79	74	61	0	0	0	-1
normalized size	1	1.00	1.18	1.10	0.91	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.082	0.072	0.524	0.743	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	30	55	0	0	-1
normalized size	1	1.00	0.92	1.32	1.20	2.20	0.00	0.00	-0.04
time (sec)	N/A	0.037	0.027	0.022	0.610	0.578	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	68	77	65	0	0	0	-1
normalized size	1	1.00	0.96	1.08	0.92	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.071	0.112	0.692	0.524	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	77	69	0	0	0	-1
normalized size	1	1.00	0.96	1.03	0.92	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.076	0.053	0.668	0.439	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	89	0	82	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	1.509	0.131	0.443	0.479	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	82	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	1.308	0.145	0.421	0.506	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	68	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	1.113	0.095	0.418	0.485	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	40	37	69	0	0	-1
normalized size	1	1.00	0.91	0.93	0.86	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.035	0.140	0.401	0.535	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	0	74	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.81	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	1.428	0.099	0.429	0.457	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	161	0	149	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.90	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	1.538	0.164	0.499	0.446	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	161	0	149	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.90	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	1.599	0.135	0.502	0.654	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	140	0	125	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	1.211	0.125	0.495	0.489	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	52	67	62	115	0	0	-1
normalized size	1	1.00	0.78	1.00	0.93	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.062	0.191	0.433	0.604	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	126	0	133	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.86	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	1.408	0.137	0.464	0.485	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.021	5.406	0.881	0.000	0.727	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	8.076	0.743	0.000	0.651	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	93	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.135	0.960	0.000	0.519	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.051	5.895	0.880	0.000	0.587	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	167	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.411	0.846	0.000	0.512	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	8.447	0.790	0.000	0.521	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	185	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	2.151	0.324	0.000	0.555	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	1.998	0.260	0.000	0.496	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	102	115	0	0	0	0	-1
normalized size	1	1.00	1.03	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.182	0.270	0.000	0.570	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	24.337	0.175	0.000	0.539	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	74	34	139	0	0	-1
normalized size	1	1.00	1.02	1.64	0.76	3.09	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.067	0.047	0.406	0.456	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	90	47	182	0	0	-1
normalized size	1	1.00	0.81	1.34	0.70	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.138	0.123	0.428	0.626	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	152	70	303	0	0	-1
normalized size	1	1.00	0.84	1.35	0.62	2.68	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.219	0.169	0.482	0.538	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	54	69	97	0	53	-1
normalized size	1	1.00	0.85	0.76	0.97	1.37	0.00	0.75	-0.01
time (sec)	N/A	0.045	1.577	0.076	0.395	0.725	0.000	0.146	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	63	136	817	165	0	137	-1
normalized size	1	1.00	0.56	1.20	7.23	1.46	0.00	1.21	-0.01
time (sec)	N/A	0.096	0.138	0.070	0.743	0.532	0.000	0.150	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	66	649	134	0	99	-1
normalized size	1	1.00	0.81	1.22	12.02	2.48	0.00	1.83	-0.02
time (sec)	N/A	0.053	0.028	0.043	0.675	0.552	0.000	0.148	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	36	477	55	0	39	-1
normalized size	1	1.00	0.73	0.97	12.89	1.49	0.00	1.05	-0.03
time (sec)	N/A	0.017	0.005	0.042	0.585	0.586	0.000	0.129	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.036	9.983	0.063	0.000	0.531	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.039	13.406	0.067	0.000	0.658	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	104	831	486	104	269	914	-1
normalized size	1	1.00	0.30	2.40	1.40	0.30	0.78	2.64	-0.00
time (sec)	N/A	0.418	1.302	0.022	0.363	0.535	1.743	0.164	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	72	303	293	68	151	299	-1
normalized size	1	1.00	0.43	1.81	1.75	0.41	0.90	1.79	-0.01
time (sec)	N/A	0.186	0.207	0.019	0.325	0.643	0.615	0.136	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	63	111	44	65	64	43
normalized size	1	1.00	0.93	1.17	2.06	0.81	1.20	1.19	0.80
time (sec)	N/A	0.046	0.063	0.008	0.320	0.591	0.475	0.136	0.441

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	130	0	0	217	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.989	0.032	0.000	0.582	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	199	0	0	315	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	1.73	0.00	0.00	-0.01
time (sec)	N/A	0.361	3.092	0.030	0.000	0.487	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	378	1815	642	180	0	2162	-1
normalized size	1	1.00	0.70	3.38	1.20	0.34	0.00	4.03	-0.00
time (sec)	N/A	0.697	2.902	0.020	0.331	0.489	0.000	0.207	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	118	659	371	109	0	706	-1
normalized size	1	1.00	0.45	2.52	1.42	0.42	0.00	2.70	-0.00
time (sec)	N/A	0.315	0.348	0.020	0.323	0.520	0.000	0.147	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	133	137	58	94	128	75
normalized size	1	1.00	0.76	1.56	1.61	0.68	1.11	1.51	0.88
time (sec)	N/A	0.079	0.086	0.017	0.318	0.564	1.216	0.132	0.467

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	233	0	0	503	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.519	0.070	0.040	0.000	0.673	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	210	0	0	704	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.724	1.849	0.035	0.000	0.644	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [88] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	0	0	0.00	0	0.000
24	A	8	3	1.00	16	0.188
25	A	5	3	1.00	16	0.188
26	A	3	2	1.00	14	0.143
27	A	0	0	0.00	0	0.000
28	A	2	2	1.00	12	0.167
29	A	7	5	1.00	12	0.417
30	A	6	5	1.00	10	0.500
31	A	5	5	1.00	8	0.625
32	A	3	3	1.00	12	0.250
33	A	2	2	1.00	12	0.167
34	A	3	3	1.00	12	0.250
35	A	4	3	1.00	12	0.250
36	A	5	3	1.00	12	0.250
37	A	9	4	1.00	16	0.250
38	A	6	4	1.00	16	0.250
39	A	4	3	1.00	14	0.214
40	A	0	0	0.00	0	0.000
41	A	7	6	1.00	12	0.500
42	A	6	5	1.00	12	0.417
43	A	6	6	1.00	12	0.500
44	A	5	5	1.00	10	0.500
45	A	5	5	1.00	8	0.625
46	A	3	3	1.00	12	0.250
47	A	4	4	1.00	12	0.333
48	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
49	A	5	5	1.00	12	0.417
50	A	3	3	1.00	12	0.250
51	A	6	6	1.00	12	0.500
52	A	4	3	1.00	12	0.250
53	A	9	4	1.00	16	0.250
54	A	6	4	1.00	16	0.250
55	A	4	3	1.00	14	0.214
56	A	0	0	0.00	0	0.000
57	A	2	2	1.00	12	0.167
58	A	3	2	1.00	12	0.167
59	A	3	2	1.00	10	0.200
60	A	3	2	1.00	8	0.250
61	A	3	3	1.00	12	0.250
62	A	3	2	1.00	12	0.167
63	A	3	2	1.00	12	0.167
64	A	5	3	1.00	14	0.214
65	A	5	3	1.00	12	0.250
66	A	5	3	1.00	10	0.300
67	A	5	4	1.00	14	0.286
68	A	5	3	1.00	14	0.214
69	A	8	3	1.00	14	0.214
70	A	8	3	1.00	12	0.250
71	A	8	3	1.00	10	0.300
72	A	8	4	1.00	14	0.286
73	A	8	3	1.00	14	0.214
74	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
75	A	0	0	0.00	0	0.000
76	A	3	3	1.00	20	0.150
77	A	0	0	0.00	0	0.000
78	A	5	5	1.00	22	0.227
79	A	0	0	0.00	0	0.000
80	A	8	3	1.00	16	0.188
81	A	5	3	1.00	16	0.188
82	A	3	2	1.00	14	0.143
83	A	0	0	0.00	0	0.000
84	A	5	5	1.00	16	0.312
85	A	7	6	1.00	18	0.333
86	A	12	6	1.00	18	0.333
87	A	4	4	1.00	18	0.222
88	A	12	9	1.00	12	0.750
89	A	8	7	1.00	10	0.700
90	A	4	4	1.00	8	0.500
91	A	0	0	0.00	0	0.000
92	A	0	0	0.00	0	0.000
93	A	16	4	1.00	18	0.222
94	A	10	4	1.00	16	0.250
95	A	4	4	1.00	14	0.286
96	A	10	5	1.00	18	0.278
97	A	11	6	1.00	18	0.333
98	A	23	6	1.00	18	0.333
99	A	13	5	1.00	16	0.312
100	A	5	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
101	A	13	5	1.00	18	0.278
102	A	14	6	1.00	18	0.333

Chapter 3

Listing of integrals

3.1 $\int x^3 \sinh\left(a + bx^2\right) dx$

Optimal. Leaf size=34

$$\frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*x^2*cosh(b*x^2+a)/b-1/2*sinh(b*x^2+a)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 3296, 2637}

$$\frac{x^2\cosh\left(a+bx^2\right)}{2b}-\frac{\sinh\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^2],x]
```

```
[Out] (x^2*Cosh[a + b*x^2])/(2*b) - Sinh[a + b*x^2]/(2*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3 \sinh\left(a+bx^2\right) dx &= \frac{1}{2}\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, x^2\right)\\ &= \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\text{Subst}\left(\int \cosh(a+bx)\,dx, x, x^2\right)}{2b}\\ &= \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\sinh\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.91

$$\frac{bx^2\cosh\left(a+bx^2\right)-\sinh\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^2],x]
[Out] (b*x^2*Cosh[a + b*x^2] - Sinh[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.50, size = 29, normalized size = 0.85

$$\frac{bx^2\cosh\left(bx^2+a\right)-\sinh\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="fricas")
[Out] 1/2*(b*x^2*cosh(b*x^2 + a) - sinh(b*x^2 + a))/b^2
```

giac [A] time = 0.26, size = 47, normalized size = 1.38

$$\frac{\frac{(bx^2-1)e^{(bx^2+a)}}{b}+\frac{(bx^2+1)e^{(-bx^2-a)}}{b}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 - 1)*e^(b*x^2 + a)/b + (b*x^2 + 1)*e^(-b*x^2 - a)/b)/b
```

maple [A] time = 0.02, size = 45, normalized size = 1.32

$$\frac{\left(b\,x^2-1\right)e^{b\,x^2+a}}{4b^2}+\frac{\left(b\,x^2+1\right)e^{-b\,x^2-a}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^2+a),x)
```

```
[Out] 1/4*(b*x^2-1)/b^2*exp(b*x^2+a)+1/4*(b*x^2+1)/b^2*exp(-b*x^2-a)
```

maxima [B] time = 0.31, size = 81, normalized size = 2.38

$$\frac{1}{4}\,x^4\sinh\left(bx^2+a\right)-\frac{1}{8}\,b\left(\frac{\left(b^2x^4e^a-2\,bx^2e^a+2\,e^a\right)e^{\left(bx^2\right)}}{b^3}-\frac{\left(b^2x^4+2\,bx^2+2\right)e^{\left(-bx^2-a\right)}}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*sinh(b*x^2 + a) - 1/8*b*((b^2*x^4*e^a - 2*b*x^2*e^a + 2*e^a)*e^(b*x
^2)/b^3 - (b^2*x^4 + 2*b*x^2 + 2)*e^(-b*x^2 - a)/b^3)
```

mupad [B] time = 0.09, size = 28, normalized size = 0.82

$$-\frac{\sinh\left(b\,x^2+a\right)-b\,x^2\cosh\left(b\,x^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a + b*x^2),x)
```

```
[Out] -(sinh(a + b*x^2) - b*x^2*cosh(a + b*x^2))/(2*b^2)
```

sympy [A] time = 0.80, size = 36, normalized size = 1.06

$$\begin{cases} \frac{x^2\cosh\left(a+bx^2\right)}{2b} - \frac{\sinh\left(a+bx^2\right)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^4\sinh\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**2+a),x)
```

```
[Out] Piecewise((x**2*cosh(a + b*x**2)/(2*b) - sinh(a + b*x**2)/(2*b**2), Ne(b, 0
)), (x**4*sinh(a)/4, True))
```

3.2 $\int x^2 \sinh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{\pi}\,e^{-a}\mathrm{erf}\left(\sqrt{b}\,x\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\,e^{a}\mathrm{erfi}\left(\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{x\cosh\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*x*cosh(b*x^2+a)/b-1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)-1/8*exp(a)
*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

Rubi [**A**] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5324, 5299, 2204, 2205}

$$-\frac{\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{x\cosh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^2],x]
```

```
[Out] (x*Cosh[a + b*x^2])/(2*b) - (Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*b^(3/2)*E^a) - (E^
a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(8*b^(3/2))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
```

```
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh\left(a+bx^2\right) dx &= \frac{x\cosh\left(a+bx^2\right)}{2b} - \frac{\int \cosh\left(a+bx^2\right) dx}{2b} \\
&= \frac{x\cosh\left(a+bx^2\right)}{2b} - \frac{\int e^{-a-bx^2} dx}{4b} - \frac{\int e^{a+bx^2} dx}{4b} \\
&= \frac{x\cosh\left(a+bx^2\right)}{2b} - \frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}\,x\right)}{8b^{3/2}} - \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}\,x\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 67, normalized size = 0.97

$$
\frac{\sqrt{\pi}(\sinh(a)-\cosh(a))\operatorname{erf}\left(\sqrt{b}\,x\right)-\sqrt{\pi}(\sinh(a)+\cosh(a))\operatorname{erfi}\left(\sqrt{b}\,x\right)+4\sqrt{b}\,x\cosh\left(a+bx^2\right)}{8b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^2],x]
```

```
[Out] (4*Sqrt[b]*x*Cosh[a + b*x^2] + Sqrt[Pi]*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a])
 - Sqrt[Pi]*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]))/(8*b^(3/2))
```

fricas [B] time = 0.52, size = 190, normalized size = 2.75

$$
\frac{2\,bx\cosh\left(bx^2+a\right)^2+4\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)+2\,bx\sinh\left(bx^2+a\right)^2+\sqrt{\pi}\left(\cosh\left(bx^2+a\right)\cosh(a)+\right.}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x*cosh(b*x^2 + a)^2 + 4*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + 2*b*
x*sinh(b*x^2 + a)^2 + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) + sinh(a
))*sinh(b*x^2 + a) + cosh(b*x^2 + a)*sinh(a))*sqrt(-b)*erf(sqrt(-b)*x) - sq
rt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a) - cos
h(b*x^2 + a)*sinh(a))*sqrt(b)*erf(sqrt(b)*x) + 2*b*x)/(b^2*cosh(b*x^2 + a)
+ b^2*sinh(b*x^2 + a))
```

giac [A] time = 0.36, size = 75, normalized size = 1.09

$$\frac{xe^{\left(bx^2+a\right)}}{4\,b}+\frac{xe^{\left(-bx^2-a\right)}}{4\,b}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{8\,b^{\frac{3}{2}}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{8\,\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*x*e^(b*x^2 + a)/b + 1/4*x*e^(-b*x^2 - a)/b + 1/8*sqrt(pi)*erf(-sqrt(b)*
x)*e^(-a)/b^(3/2) + 1/8*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

maple [A] time = 0.08, size = 74, normalized size = 1.07

$$\frac{\mathrm{e}^{-a}x\,\mathrm{e}^{-b\,x^2}}{4b}-\frac{\mathrm{e}^{-a}\sqrt{\pi}\ \operatorname{erf}\left(x\sqrt{b}\right)}{8b^{\frac{3}{2}}}+\frac{\mathrm{e}^{a}\mathrm{e}^{b\,x^2}x}{4b}-\frac{\mathrm{e}^{a}\sqrt{\pi}\ \operatorname{erf}\left(\sqrt{-b}\ x\right)}{8b\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(b*x^2+a),x)
```

```
[Out] 1/4*exp(-a)/b*x*exp(-b*x^2)-1/8*exp(-a)/b^(3/2)*Pi^(1/2)*erf(x*b^(1/2))+1/4
*exp(a)*exp(b*x^2)*x/b-1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

maxima [B] time = 0.30, size = 110, normalized size = 1.59

$$\frac{1}{3}\,x^3\sinh\left(bx^2+a\right)-\frac{1}{24}\,b\left(\frac{2\left(2\,bx^3e^a-3\,xe^a\right)e^{\left(bx^2\right)}}{b^2}-\frac{2\left(2\,bx^3+3\,x\right)e^{\left(-bx^2-a\right)}}{b^2}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{b^{\frac{5}{2}}}+\frac{3\,\sqrt{\pi}\,\operatorname{er}}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(b*x^2 + a) - 1/24*b*(2*(2*b*x^3*e^a - 3*x*e^a)*e^(b*x^2)/b^2 -
 2*(2*b*x^3 + 3*x)*e^(-b*x^2 - a)/b^2 + 3*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^
(5/2) + 3*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sinh\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*x^2),x)
```

```
[Out] int(x^2*sinh(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(b*x**2+a),x)
[Out] Integral(x**2*sinh(a + b*x**2), x)
```

3.3 $\int x \sinh\left(a + bx^2\right) dx$

Optimal. Leaf size=15

$$\frac{\cosh\left(a + bx^2\right)}{2b}$$

```
[Out] 1/2*cosh(b*x^2+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5320, 2638}

$$\frac{\cosh\left(a + bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2],x]
```

```
[Out] Cosh[a + b*x^2]/(2*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x \sinh\left(a + bx^2\right) dx &= \frac{1}{2}\,\text{Subst}\left(\int \sinh(a + bx)\, dx, x, x^2\right) \\
&= \frac{\cosh\left(a + bx^2\right)}{2b}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 31, normalized size = 2.07

$$\frac{\sinh(a)\sinh\left(bx^2\right)}{2b} + \frac{\cosh(a)\cosh\left(bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2],x]
```

```
[Out] (Cosh[a]*Cosh[b*x^2])/(2*b) + (Sinh[a]*Sinh[b*x^2])/(2*b)
```

fricas [A] time = 0.72, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*cosh(b*x^2 + a)/b
```

giac [A] time = 0.39, size = 25, normalized size = 1.67

$$\frac{e^{\left(bx^2+a\right)} + e^{\left(-bx^2-a\right)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(e^(b*x^2 + a) + e^(-b*x^2 - a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\cosh\left(b\,x^2 + a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a),x)
```

```
[Out] 1/2*cosh(b*x^2+a)/b
```

maxima [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*cosh(b*x^2 + a)/b
```

mupad [B] time = 0.37, size = 13, normalized size = 0.87

$$\frac{\cosh\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^2),x)
```

```
[Out] cosh(a + b*x^2)/(2*b)
```

sympy [A] time = 0.21, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\cosh\left(a+bx^2\right)}{2b} & \text{for}\ b \neq 0 \\ \frac{x^2\sinh\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a),x)
```

```
[Out] Piecewise((cosh(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*sinh(a)/2, True))
```

3.4 $\int \sinh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\pi}\,e^{a}\operatorname{erfi}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,e^{-a}\operatorname{erf}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}$$

```
[Out] -1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*erfi(x*b^(1/2))*Pi^(
1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5298, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^{a}\operatorname{Erfi}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,e^{-a}\operatorname{Erf}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[b]*x])/(4*Sqrt[b]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])
/(4*Sqrt[b])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\int \sinh\left(a+bx^2\right)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^2}\,dx\right)+\frac{1}{2}\int e^{a+bx^2}\,dx$$
$$= -\frac{e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}+\frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)}{4\sqrt{b}}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.85

$$\frac{\sqrt{\pi}\,\left((\sinh(a)-\cosh(a))\mathrm{erf}\left(\sqrt{b}\,x\right)+(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\sqrt{b}\,x\right)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Erfi[Sqrt[b]*x]*(Cosh[a] +
 Sinh[a])))/(4*Sqrt[b])
```

fricas [A] time = 0.46, size = 48, normalized size = 0.91

$$-\frac{\sqrt{\pi}\,\sqrt{-b}\,(\cosh(a)+\sinh(a))\,\mathrm{erf}\left(\sqrt{-b}\,x\right)+\sqrt{\pi}\,\sqrt{b}\,(\cosh(a)-\sinh(a))\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) + sqrt(pi)*sqrt
(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

giac [A] time = 0.30, size = 41, normalized size = 0.77

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{4\,\sqrt{b}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{4\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 1/4*sqrt(pi)*erf(-sqrt(-b)*x)
*e^a/sqrt(-b)
```

maple [A] time = 0.03, size = 40, normalized size = 0.75

$$-\frac{\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\ \mathrm{e}^{-a}}{4\sqrt{b}}+\frac{\mathrm{e}^{a}\sqrt{\pi}\ \operatorname{erf}\left(\sqrt{-b}\ x\right)}{4\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a),x)
```

```
[Out] -1/4*erf(x*b^(1/2))*Pi^(1/2)*exp(-a)/b^(1/2)+1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)
*erf((-b)^(1/2)*x)
```

maxima [B] time = 0.30, size = 86, normalized size = 1.62

$$-\frac{1}{4}\,b\left(\frac{2\,xe^{\left(bx^2+a\right)}}{b}-\frac{2\,xe^{\left(-bx^2-a\right)}}{b}+\frac{\sqrt{\pi}\ \operatorname{erf}\left(\sqrt{b}\,x\right)e^{\left(-a\right)}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\ \operatorname{erf}\left(\sqrt{-b}\,x\right)e^{a}}{\sqrt{-b}\,b}\right)+x\sinh\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*x*e^(b*x^2 + a)/b - 2*x*e^(-b*x^2 - a)/b + sqrt(pi)*erf(sqrt(b)*x
)*e^(-a)/b^(3/2) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)) + x*sinh(b*x^
2 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sinh\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2),x)
```

```
[Out] int(sinh(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a),x)
```

```
[Out] Integral(sinh(a + b*x**2), x)
```

3.5 $\int \frac{\sinh\left(a+bx^2\right)}{x} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sinh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\cosh(a)\text{Shi}\left(bx^2\right)$$

```
[Out] 1/2*cosh(a)*Shi(b*x^2)+1/2*Chi(b*x^2)*sinh(a)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5318, 5317, 5316}

$$\frac{1}{2}\sinh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}\cosh(a)\text{Shi}\left(bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]/x,x]
```

```
[Out] (CoshIntegral[b*x^2]*Sinh[a])/2 + (Cosh[a]*SinhIntegral[b*x^2])/2
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+bx^2\right)}{x} \, dx &= \cosh(a)\int \frac{\sinh\left(bx^2\right)}{x} \, dx+\sinh(a)\int \frac{\cosh\left(bx^2\right)}{x} \, dx\\ &= \frac{1}{2}\text{Chi}\left(bx^2\right)\sinh(a)+\frac{1}{2}\cosh(a)\text{Shi}\left(bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(\sinh(a)\text{Chi}\left(bx^2\right)+\cosh(a)\text{Shi}\left(bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]/x,x]
[Out] (CoshIntegral[b*x^2]*Sinh[a] + Cosh[a]*SinhIntegral[b*x^2])/2
```

fricas [A] time = 0.48, size = 39, normalized size = 1.56

$$\frac{1}{4}\left(\text{Ei}\left(bx^2\right)-\text{Ei}\left(-bx^2\right)\right)\cosh(a)+\frac{1}{4}\left(\text{Ei}\left(bx^2\right)+\text{Ei}\left(-bx^2\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="fricas")
[Out] 1/4*(Ei(b*x^2) - Ei(-b*x^2))*cosh(a) + 1/4*(Ei(b*x^2) + Ei(-b*x^2))*sinh(a)
```

giac [A] time = 0.31, size = 24, normalized size = 0.96

$$-\frac{1}{4}\,\text{Ei}\left(-bx^2\right)e^{(-a)}+\frac{1}{4}\,\text{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="giac")
[Out] -1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

maple [A] time = 0.02, size = 27, normalized size = 1.08

$$\frac{\mathrm{e}^{-a}\,\text{Ei}\left(1,b\,x^2\right)}{4}-\frac{\mathrm{e}^{a}\,\text{Ei}\left(1,-b\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)/x,x)
[Out] 1/4*exp(-a)*Ei(1,b*x^2)-1/4*exp(a)*Ei(1,-b*x^2)
```

maxima [A] time = 0.37, size = 24, normalized size = 0.96

$$-\frac{1}{4}\,\text{Ei}\left(-bx^2\right)e^{(-a)}+\frac{1}{4}\,\text{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] -1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\frac{\sinh(a)\operatorname{coshint}\left(b\,x^2\right)}{2} + \frac{\cosh(a)\operatorname{sinhint}\left(b\,x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)/x,x)
```

```
[Out] (sinh(a)*coshint(b*x^2))/2 + (cosh(a)*sinhint(b*x^2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)/x,x)
```

```
[Out] Integral(sinh(a + b*x**2)/x, x)
```

3.6 $\int \frac{\sinh(a+bx^2)}{x^2} dx$

Optimal. Leaf size=66

$$\frac{1}{2}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{erf}\left(\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)-\frac{\sinh\left(a+bx^2\right)}{x}$$

```
[Out] -sinh(b*x^2+a)/x+1/2*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)+1/2*exp(a)*erfi
(x*b^(1/2))*b^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5326, 5299, 2204, 2205}

$$\frac{1}{2}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\sqrt{b}\,x\right)-\frac{\sinh\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(2*E^a) + (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt
[b]*x])/2 - Sinh[a + b*x^2]/x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sinh\left(a+bx^2\right)}{x}+(2b)\int \cosh\left(a+bx^2\right)\,dx\\ &= -\frac{\sinh\left(a+bx^2\right)}{x}+b\int e^{-a-bx^2}\,dx+b\int e^{a+bx^2}\,dx\\ &= \frac{1}{2}\sqrt{b}\,e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{b}\,e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}\,x\right)-\frac{\sinh\left(a+bx^2\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 1.06

$$\frac{\sqrt{\pi}\sqrt{b}\,x(\cosh(a)-\sinh(a))\operatorname{erf}\left(\sqrt{b}\,x\right)+\sqrt{\pi}\sqrt{b}\,x(\sinh(a)+\cosh(a))\operatorname{erfi}\left(\sqrt{b}\,x\right)-2\sinh\left(a+bx^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]/x^2,x]
```

```
[Out] (Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + Sqrt[b]*Sqrt[Pi]*x
*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]) - 2*Sinh[a + b*x^2])/(2*x)
```

fricas [B] time = 0.48, size = 184, normalized size = 2.79

$$-\frac{\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)\cosh(a)+x\cosh\left(bx^2+a\right)\sinh(a)+(x\cosh(a)+x\sinh(a))\sinh\left(bx^2+a\right)\right)\sqrt{-b}\operatorname{erf}\left(\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a) + (x*
cosh(a) + x*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) - sqrt(pi)*(
x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a) + (x*cosh(a) - x*sinh
(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + cosh(b*x^2 + a)^2 + 2*cosh(b
*x^2 + a)*sinh(b*x^2 + a) + sinh(b*x^2 + a)^2 - 1)/(x*cosh(b*x^2 + a) + x*s
inh(b*x^2 + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx^2+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^2 + a)/x^2, x)
```

maple [A] time = 0.03, size = 70, normalized size = 1.06

$$\frac{e^{-a}e^{-b\,x^2}}{2x}+\frac{e^{-a}\sqrt{b}\,\sqrt{\pi}\,\operatorname{erf}\left(x\sqrt{b}\right)}{2}-\frac{e^{a}e^{b\,x^2}}{2x}+\frac{e^{a}b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)/x^2,x)
```

```
[Out] 1/2*exp(-a)/x*exp(-b*x^2)+1/2*exp(-a)*b^(1/2)*Pi^(1/2)*erf(x*b^(1/2))-1/2*e
xp(a)*exp(b*x^2)/x+1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

maxima [A] time = 0.32, size = 54, normalized size = 0.82

$$\frac{1}{2}\left(\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{\sqrt{b}}+\frac{\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)e^{a}}{\sqrt{-b}}\right)b-\frac{\sinh\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) + sqrt(pi)*erf(sqrt(-b)*x)*e^a/
sqrt(-b))*b - sinh(b*x^2 + a)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(b\,x^2+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)/x^2,x)
```

```
[Out] int(sinh(a + b*x^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh \left(a + bx^2\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)/x**2, x)
```

3.7 $\int \frac{\sinh(a+bx^2)}{x^3} dx$

Optimal. Leaf size=42

$$\frac{1}{2}b\cosh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}b\sinh(a)\text{Shi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{2x^2}$$

```
[Out] 1/2*b*Chi(b*x^2)*cosh(a)+1/2*b*Shi(b*x^2)*sinh(a)-1/2*sinh(b*x^2+a)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5320, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\cosh(a)\text{Chi}\left(bx^2\right)+\frac{1}{2}b\sinh(a)\text{Shi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]/x^3,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2])/2 - Sinh[a + b*x^2]/(2*x^2) + (b*Sinh[a]*Si
nhIntegral[b*x^2])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,x^2\right)\\ &= -\frac{\sinh\left(a+bx^2\right)}{2x^2}+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,x^2\right)\\ &= -\frac{\sinh\left(a+bx^2\right)}{2x^2}+\frac{1}{2}(b\cosh(a))\operatorname{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,x^2\right)+\frac{1}{2}(b\sinh(a))\operatorname{Subst}\left(\int\right.\\ &= \frac{1}{2}b\cosh(a)\operatorname{Chi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{2x^2}+\frac{1}{2}b\sinh(a)\operatorname{Shi}\left(bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.90

$$\frac{1}{2}\left(b\cosh(a)\operatorname{Chi}\left(bx^2\right)+b\sinh(a)\operatorname{Shi}\left(bx^2\right)-\frac{\sinh\left(a+bx^2\right)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]/x^3,x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^2] - Sinh[a + b*x^2]/x^2 + b*Sinh[a]*SinhIntegr
al[b*x^2])/2
```

fricas [A] time = 0.46, size = 71, normalized size = 1.69

$$\frac{\left(bx^2\operatorname{Ei}\left(bx^2\right)+bx^2\operatorname{Ei}\left(-bx^2\right)\right)\cosh(a)+\left(bx^2\operatorname{Ei}\left(bx^2\right)-bx^2\operatorname{Ei}\left(-bx^2\right)\right)\sinh(a)-2\,\sinh\left(bx^2+a\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*cosh(a) + (b*x^2*Ei(b*x^2) - b*x^
2*Ei(-b*x^2))*sinh(a) - 2*sinh(b*x^2 + a))/x^2
```

giac [B] time = 0.64, size = 109, normalized size = 2.60

$$\frac{\left(bx^2+a\right)b^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)}-ab^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+\left(bx^2+a\right)b^2\mathrm{Ei}\left(bx^2\right)e^{a}-ab^2\mathrm{Ei}\left(bx^2\right)e^{a}-b^2e^{\left(bx^2+a\right)}+b^2e^{\left(-bx^2-a\right)}}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) - a*b^2*Ei(-b*x^2)*e^(-a) + (b*x^2 +
 a)*b^2*Ei(b*x^2)*e^a - a*b^2*Ei(b*x^2)*e^a - b^2*e^(b*x^2 + a) + b^2*e^(-b
*x^2 - a))/(b^2*x^2)
```

maple [A] time = 0.02, size = 58, normalized size = 1.38

$$\frac{\mathrm{e}^{-a}\mathrm{e}^{-b\,x^2}}{4x^2}-\frac{\mathrm{e}^{-a}b\,\mathrm{Ei}\left(1,b\,x^2\right)}{4}-\frac{\mathrm{e}^{a}\mathrm{e}^{b\,x^2}}{4x^2}-\frac{\mathrm{e}^{a}b\,\mathrm{Ei}\left(1,-b\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)/x^3,x)
```

```
[Out] 1/4*exp(-a)/x^2*exp(-b*x^2)-1/4*exp(-a)*b*Ei(1,b*x^2)-1/4*exp(a)*exp(b*x^2)
/x^2-1/4*exp(a)*b*Ei(1,-b*x^2)
```

maxima [A] time = 0.43, size = 39, normalized size = 0.93

$$\frac{1}{4}\left(\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+\mathrm{Ei}\left(bx^2\right)e^{a}\right)b-\frac{\sinh\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(Ei(-b*x^2)*e^(-a) + Ei(b*x^2)*e^a)*b - 1/2*sinh(b*x^2 + a)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(b\,x^2+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)/x^3,x)
```

```
[Out] int(sinh(a + b*x^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + bx^2\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)/x**3, x)
```

3.8 $\int x^3 \sinh^2\left(a+bx^2\right) dx$

Optimal. Leaf size=51

$$-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^4}{8}$$

```
[Out] -1/8*x^4+1/4*x^2*cosh(b*x^2+a)*sinh(b*x^2+a)/b-1/8*sinh(b*x^2+a)^2/b^2
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5320, 3310, 30}

$$-\frac{\sinh^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^2]^2,x]
```

```
[Out] -x^4/8 + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b) - Sinh[a + b*x^2]^2/(8
*b^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sinh^2\left(a+bx^2\right) dx = \frac{1}{2}\operatorname{Subst}\left(\int x\sinh^2(a+bx)\,dx, x, x^2\right)$$
$$= \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} - \frac{\sinh^2\left(a+bx^2\right)}{8b^2} - \frac{1}{4}\operatorname{Subst}\left(\int x\,dx, x, x^2\right)$$
$$= -\frac{x^4}{8} + \frac{x^2\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b} - \frac{\sinh^2\left(a+bx^2\right)}{8b^2}$$

Mathematica [A] time = 0.10, size = 42, normalized size = 0.82

$$-\frac{2bx^2\left(bx^2-\sinh\left(2\left(a+bx^2\right)\right)\right)+\cosh\left(2\left(a+bx^2\right)\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^2]^2,x]
[Out] -1/16*(Cosh[2*(a + b*x^2)] + 2*b*x^2*(b*x^2 - Sinh[2*(a + b*x^2)]))/b^2
```

fricas [A] time = 0.48, size = 56, normalized size = 1.10

$$-\frac{2\,b^2x^4-4\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)+\cosh\left(bx^2+a\right)^2+\sinh\left(bx^2+a\right)^2}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/16*(2*b^2*x^4 - 4*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + cosh(b*x^2 + a
)^2 + sinh(b*x^2 + a)^2)/b^2
```

giac [B] time = 0.54, size = 117, normalized size = 2.29

$$-\frac{4\left(bx^2+a\right)^2-8\left(bx^2+a\right)a-2\left(bx^2+a\right)e^{\left(2\,bx^2+2\,a\right)}+2\,ae^{\left(2\,bx^2+2\,a\right)}+2\left(bx^2+a\right)e^{\left(-2\,bx^2-2\,a\right)}-2\,ae^{\left(-2\,bx^2-2\,a\right)}+e}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="giac")
[Out] -1/32*(4*(b*x^2 + a)^2 - 8*(b*x^2 + a)*a - 2*(b*x^2 + a)*e^(2*b*x^2 + 2*a)
+ 2*a*e^(2*b*x^2 + 2*a) + 2*(b*x^2 + a)*e^(-2*b*x^2 - 2*a) - 2*a*e^(-2*b*x^
2 - 2*a) + e^(2*b*x^2 + 2*a) + e^(-2*b*x^2 - 2*a))/b^2
```

maple [A] time = 0.06, size = 55, normalized size = 1.08

$$-\frac{x^4}{8}+\frac{\left(2b\,x^2-1\right)\mathrm{e}^{2b\,x^2+2a}}{32b^2}-\frac{\left(2b\,x^2+1\right)\mathrm{e}^{-2b\,x^2-2a}}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^2+a)^2,x)
```

```
[Out] -1/8*x^4+1/32*(2*b*x^2-1)/b^2*exp(2*b*x^2+2*a)-1/32*(2*b*x^2+1)/b^2*exp(-2*
b*x^2-2*a)
```

maxima [A] time = 0.33, size = 59, normalized size = 1.16

$$-\frac{1}{8}\,x^4+\frac{\left(2\,bx^2e^{(2\,a)}-e^{(2\,a)}\right)e^{\left(2\,bx^2\right)}}{32\,b^2}-\frac{\left(2\,bx^2+1\right)e^{\left(-2\,bx^2-2\,a\right)}}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*x^4 + 1/32*(2*b*x^2*e^(2*a) - e^(2*a))*e^(2*b*x^2)/b^2 - 1/32*(2*b*x^2
 + 1)*e^(-2*b*x^2 - 2*a)/b^2
```

mupad [B] time = 0.10, size = 42, normalized size = 0.82

$$-\frac{\frac{\cosh\left(2\,b\,x^2+2\,a\right)}{16}-\frac{b\,x^2\,\sinh\left(2\,b\,x^2+2\,a\right)}{8}}{b^2}-\frac{x^4}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a + b*x^2)^2,x)
```

```
[Out] - (cosh(2*a + 2*b*x^2)/16 - (b*x^2*sinh(2*a + 2*b*x^2))/8)/b^2 - x^4/8
```

sympy [A] time = 1.55, size = 78, normalized size = 1.53

$$\begin{cases} \frac{x^4\sinh^2\left(a+bx^2\right)}{8}-\frac{x^4\cosh^2\left(a+bx^2\right)}{8}+\frac{x^2\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{\cosh^2\left(a+bx^2\right)}{8b^2} & \text{for } b\neq 0\\ \frac{x^4\sinh^2(a)}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**4*sinh(a + b*x**2)**2/8 - x**4*cosh(a + b*x**2)**2/8 + x**2*s
inh(a + b*x**2)*cosh(a + b*x**2)/(4*b) - cosh(a + b*x**2)**2/(8*b**2), Ne(b
, 0)), (x**4*sinh(a)**2/4, True))
```

3.9 $\int x^2 \sinh^2\left(a+bx^2\right) dx$

Optimal. Leaf size=99

$$\frac{\sqrt{\frac{\pi}{2}}\, e^{-2a}\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\, e^{2a}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}} + \frac{x\sinh\left(2a+2bx^2\right)}{8b} - \frac{x^3}{6}$$

```
[Out] -1/6*x^3+1/8*x*sinh(2*b*x^2+2*a)/b+1/64*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/exp(2*a)-1/64*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2
)/b^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5340, 5325, 5298, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\, e^{-2a}\operatorname{Erf}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\, e^{2a}\operatorname{Erfi}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}} + \frac{x\sinh\left(2a+2bx^2\right)}{8b} - \frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^2]^2,x]
```

```
[Out] -x^3/6 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)*E^(2*a)) - (E^(2*a
)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(32*b^(3/2)) + (x*Sinh[2*a + 2*b*x^2]
)/(8*b)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh^2\left(a+bx^2\right) dx &= \int \left(-\frac{x^2}{2}+\frac{1}{2}x^2\cosh\left(2a+2bx^2\right)\right) dx \\
&= -\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh\left(2a+2bx^2\right) dx \\
&= -\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}-\frac{\int \sinh\left(2a+2bx^2\right) dx}{8b} \\
&= -\frac{x^3}{6}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}+\frac{\int e^{-2a-2bx^2} dx}{16b}-\frac{\int e^{2a+2bx^2} dx}{16b} \\
&= -\frac{x^3}{6}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)}{32b^{3/2}}+\frac{x\sinh\left(2a+2bx^2\right)}{8b}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 101, normalized size = 1.02

$$\frac{3\sqrt{2\pi}(\cosh(2a)-\sinh(2a))\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)-3\sqrt{2\pi}(\sinh(2a)+\cosh(2a))\operatorname{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)+8\sqrt{b}\,x\left(3\sinh\left(2\left(a+\right.\right.\right.}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^2]^2,x]
```

```
[Out] (3*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*a]) - 3*Sqrt[2*Pi]
*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) + 8*Sqrt[b]*x*(-4*b*x^2 +
3*Sinh[2*(a + b*x^2)]))/(192*b^(3/2))
```

fricas [B] time = 0.44, size = 427, normalized size = 4.31

$$-\frac{32\,b^2x^3\cosh\left(bx^2+a\right)^2-12\,bx\cosh\left(bx^2+a\right)^4-48\,bx\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3-12\,bx\sinh\left(bx^2+a\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/192*(32*b^2*x^3*cosh(b*x^2 + a)^2 - 12*b*x*cosh(b*x^2 + a)^4 - 48*b*x*co
sh(b*x^2 + a)*sinh(b*x^2 + a)^3 - 12*b*x*sinh(b*x^2 + a)^4 - 3*sqrt(2)*sqrt
(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) + sinh(2*a))*sinh(b*x^2 + a)
^2 + cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) + cosh(b*x^
2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(2)*sqrt(-b)*x) - 3*sqr
t(2)*sqrt(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) - sinh(2*a))*sinh(b
*x^2 + a)^2 - cosh(b*x^2 + a)^2*sinh(2*a) + 2*(cosh(b*x^2 + a)*cosh(2*a) -
cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x)
+ 8*(4*b^2*x^3 - 9*b*x*cosh(b*x^2 + a)^2)*sinh(b*x^2 + a)^2 + 12*b*x + 16*(
4*b^2*x^3*cosh(b*x^2 + a) - 3*b*x*cosh(b*x^2 + a)^3)*sinh(b*x^2 + a))/(b^2*
cosh(b*x^2 + a)^2 + 2*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) + b^2*sinh(b*x^2
+ a)^2)
```

giac [A] time = 0.91, size = 97, normalized size = 0.98

$$-\frac{1}{6}\,x^3+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{2}\,\sqrt{-b}\,x\right)e^{(2\,a)}}{64\,\sqrt{-b}\,b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{2}\,\sqrt{b}\,x\right)e^{(-2\,a)}}{64\,b^{\frac{3}{2}}}+\frac{xe^{\left(2\,bx^2+2\,a\right)}}{16\,b}-\frac{xe^{\left(-2\,bx^2-2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/6*x^3 + 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)
*b) - 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16
*x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

maple [A] time = 0.09, size = 90, normalized size = 0.91

$$-\frac{x^3}{6}-\frac{\mathrm{e}^{-2a}x\,\mathrm{e}^{-2b\,x^2}}{16b}+\frac{\mathrm{e}^{-2a}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{64b^{\frac{3}{2}}}+\frac{\mathrm{e}^{2a}x\,\mathrm{e}^{2b\,x^2}}{16b}-\frac{\mathrm{e}^{2a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2b}\,x\right)}{32b\sqrt{-2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(b*x^2+a)^2,x)
```

```
[Out] -1/6*x^3-1/16*exp(-2*a)/b*x*exp(-2*b*x^2)+1/64*exp(-2*a)/b^(3/2)*Pi^(1/2)*2
^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/16*exp(2*a)/b*x*exp(2*b*x^2)-1/32*exp(2*a)/
b*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

maxima [A] time = 0.43, size = 95, normalized size = 0.96

$$-\frac{1}{6}x^3 - \frac{\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\sqrt{-b}\,x\right)e^{(2a)}}{64\sqrt{-b}\,b} + \frac{\sqrt{2}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\sqrt{b}\,x\right)e^{(-2a)}}{64\,b^{\frac{3}{2}}} + \frac{xe^{\left(2bx^2+2a\right)}}{16\,b} - \frac{xe^{\left(-2bx^2-2a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^3 - 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*
b) + 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*x
*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sinh\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*x^2)^2,x)
```

```
[Out] int(x^2*sinh(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh^2\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2)**2, x)
```

3.10 $\int x \sinh^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=31

$$\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^2}{4}$$

```
[Out] -1/4*x^2+1/4*cosh(b*x^2+a)*sinh(b*x^2+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 2635, 8}

$$\frac{\sinh\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{4b}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2]^2,x]
```

```
[Out] -x^2/4 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x\sinh^2\left(a+bx^2\right)\,dx=\frac{1}{2}\text{Subst}\left(\int\sinh^2(a+bx)\,dx,x,x^2\right)$$
$$=\frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}-\frac{1}{4}\text{Subst}\left(\int 1\,dx,x,x^2\right)$$
$$=-\frac{x^2}{4}+\frac{\cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)}{4b}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.87

$$\frac{\sinh\left(2\left(a+bx^2\right)\right)-2\left(a+bx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2]^2,x]
[Out] (-2*(a + b*x^2) + Sinh[2*(a + b*x^2)])/(8*b)
```

fricas [A] time = 0.52, size = 29, normalized size = 0.94

$$-\frac{bx^2-\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="fricas")
[Out] -1/4*(b*x^2 - cosh(b*x^2 + a)*sinh(b*x^2 + a))/b
```

giac [B] time = 0.79, size = 56, normalized size = 1.81

$$-\frac{4\,bx^2-\left(2\,e^{\left(2\,bx^2+2\,a\right)}-1\right)e^{\left(-2\,bx^2-2\,a\right)}+4\,a-e^{\left(2\,bx^2+2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="giac")
[Out] -1/16*(4*b*x^2 - (2*e^(2*b*x^2 + 2*a) - 1)*e^(-2*b*x^2 - 2*a) + 4*a - e^(2*
b*x^2 + 2*a))/b
```

maple [A] time = 0.01, size = 34, normalized size = 1.10

$$\frac{\frac{\cosh\left(b\,x^{2}+a\right)\sinh\left(b\,x^{2}+a\right)}{2}-\frac{b\,x^{2}}{2}-\frac{a}{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a)^2,x)
```

```
[Out] 1/2/b*(1/2*cosh(b*x^2+a)*sinh(b*x^2+a)-1/2*b*x^2-1/2*a)
```

maxima [A] time = 0.33, size = 38, normalized size = 1.23

$$-\frac{1}{4}\,x^{2}+\frac{e^{\left(2\,bx^{2}+2\,a\right)}}{16\,b}-\frac{e^{\left(-2\,bx^{2}-2\,a\right)}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/16*e^(2*b*x^2 + 2*a)/b - 1/16*e^(-2*b*x^2 - 2*a)/b
```

mupad [B] time = 0.38, size = 22, normalized size = 0.71

$$\frac{\sinh\left(2\,b\,x^{2}+2\,a\right)}{8\,b}-\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^2)^2,x)
```

```
[Out] sinh(2*a + 2*b*x^2)/(8*b) - x^2/4
```

sympy [A] time = 0.44, size = 60, normalized size = 1.94

$$\begin{cases} \frac{x^{2}\sinh^{2}\left(a+bx^{2}\right)}{4}-\frac{x^{2}\cosh^{2}\left(a+bx^{2}\right)}{4}+\frac{\sinh\left(a+bx^{2}\right)\cosh\left(a+bx^{2}\right)}{4b} & \text{for}\ b\neq 0\\ \frac{x^{2}\sinh^{2}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x**2)**2/4 - x**2*cosh(a + b*x**2)**2/4 + sinh(a
 + b*x**2)*cosh(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*sinh(a)**2/2, True))
```

3.11 $\int \sinh^2\left(a+bx^2\right) dx$

Optimal. Leaf size=78

$$\frac{\sqrt{\frac{\pi}{2}}\, e^{-2a} \operatorname{erf}\left(\sqrt{2}\sqrt{b}\, x\right)}{8\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\, e^{2a} \operatorname{erfi}\left(\sqrt{2}\sqrt{b}\, x\right)}{8\sqrt{b}} - \frac{x}{2}$$

```
[Out] -1/2*x+1/16*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a)/b^(1/2)+1/16*e
xp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5300, 5299, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\, e^{-2a} \operatorname{Erf}\left(\sqrt{2}\sqrt{b}\, x\right)}{8\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}}\, e^{2a} \operatorname{Erfi}\left(\sqrt{2}\sqrt{b}\, x\right)}{8\sqrt{b}} - \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2,x]
```

```
[Out] -x/2 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)*S
qrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5300

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2\left(a+bx^2\right)\,dx &= \int \left(-\frac{1}{2}+\frac{1}{2}\cosh\left(2a+2bx^2\right)\right)\,dx\\
&= -\frac{x}{2}+\frac{1}{2}\int \cosh\left(2a+2bx^2\right)\,dx\\
&= -\frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^2}\,dx+\frac{1}{4}\int e^{2a+2bx^2}\,dx\\
&= -\frac{x}{2}+\frac{e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}\,x\right)}{8\sqrt{b}}+\frac{e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 1.10

$$\frac{\sqrt{\pi}(\cosh(2a)-\sinh(2a))\mathrm{erf}\left(\sqrt{2}\sqrt{b}\,x\right)+\sqrt{\pi}(\sinh(2a)+\cosh(2a))\mathrm{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)-4\sqrt{2}\sqrt{b}\,x}{8\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2,x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[b]*x + Sqrt[Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2
*a]) + Sqrt[Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]))/(8*Sqrt[2]
*Sqrt[b])
```

fricas [A] time = 0.44, size = 73, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt{\pi}\sqrt{-b}\left(\cosh\left(2\,a\right)+\sinh\left(2\,a\right)\right)\mathrm{erf}\left(\sqrt{2}\sqrt{-b}\,x\right)-\sqrt{2}\sqrt{\pi}\sqrt{b}\left(\cosh\left(2\,a\right)-\sinh\left(2\,a\right)\right)\mathrm{erf}\left(\sqrt{2}\sqrt{b}\,x\right)+8\,b}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*sqrt(-b)*(cosh(2*a) + sinh(2*a))*erf(sqrt(2)*sqrt(-
b)*x) - sqrt(2)*sqrt(pi)*sqrt(b)*(cosh(2*a) - sinh(2*a))*erf(sqrt(2)*sqrt(b
)*x) + 8*b*x)/b
```

giac [A] time = 0.56, size = 58, normalized size = 0.74

$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{16\sqrt{-b}}-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{16\sqrt{b}}-\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) - 1/16*sqr
t(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) - 1/2*x
```

maple [A] time = 0.05, size = 51, normalized size = 0.65

$$-\frac{x}{2}+\frac{e^{-2a}\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{16\sqrt{b}}+\frac{e^{2a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2b}\,x\right)}{8\sqrt{-2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2,x)
```

```
[Out] -1/2*x+1/16*exp(-2*a)*Pi^(1/2)*2^(1/2)/b^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/8*e
xp(2*a)*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

maxima [A] time = 0.43, size = 56, normalized size = 0.72

$$\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{-b}x\right)e^{(2a)}}{16\sqrt{-b}}+\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{b}x\right)e^{(-2a)}}{16\sqrt{b}}-\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) + 1/16*sqrt(
2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) - 1/2*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^2,x)
```

```
[Out] int(sinh(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^2\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2, x)
```

3.12 $\int \frac{\sinh^2(a+bx^2)}{x} dx$

Optimal. Leaf size=37

$$\frac{1}{4}\cosh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{4}\sinh(2a)\text{Shi}\left(2bx^2\right)-\frac{\log(x)}{2}$$

```
[Out] 1/4*Chi(2*b*x^2)*cosh(2*a)-1/2*ln(x)+1/4*Shi(2*b*x^2)*sinh(2*a)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5340, 5319, 5317, 5316}

$$\frac{1}{4}\cosh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{4}\sinh(2a)\text{Shi}\left(2bx^2\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2])/4 - Log[x]/2 + (Sinh[2*a]*SinhIntegral[2*
b*x^2])/4
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5319

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^2\left(a+bx^2\right)}{x}\,dx = \int \left(-\frac{1}{2x}+\frac{\cosh\left(2a+2bx^2\right)}{2x}\right)dx$$
$$= -\frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^2\right)}{x}\,dx$$
$$= -\frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh\left(2bx^2\right)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh\left(2bx^2\right)}{x}\,dx$$
$$= \frac{1}{4}\cosh(2a)\text{Chi}\left(2bx^2\right)-\frac{\log(x)}{2}+\frac{1}{4}\sinh(2a)\text{Shi}\left(2bx^2\right)$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{1}{4}\left(\cosh(2a)\text{Chi}\left(2bx^2\right)+\sinh(2a)\text{Shi}\left(2bx^2\right)-2\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2] - 2*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^
2])/4
```

fricas [A] time = 0.41, size = 49, normalized size = 1.32

$$\frac{1}{8}\left(\text{Ei}\left(2\,bx^2\right)+\text{Ei}\left(-2\,bx^2\right)\right)\cosh\left(2\,a\right)+\frac{1}{8}\left(\text{Ei}\left(2\,bx^2\right)-\text{Ei}\left(-2\,bx^2\right)\right)\sinh\left(2\,a\right)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(2*b*x^2) + Ei(-2*b*x^2))*cosh(2*a) + 1/8*(Ei(2*b*x^2) - Ei(-2*b*x^2
))*sinh(2*a) - 1/2*log(x)
```

giac [A] time = 0.34, size = 35, normalized size = 0.95

$$\frac{1}{8}\,\text{Ei}\left(2\,bx^2\right)e^{(2\,a)}+\frac{1}{8}\,\text{Ei}\left(-2\,bx^2\right)e^{(-2\,a)}-\frac{1}{4}\,\log\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) - 1/4*log(b*x^2)
```

maple [A] time = 0.06, size = 34, normalized size = 0.92

$$-\frac{\ln(x)}{2} - \frac{e^{-2a}\,\mathrm{Ei}\left(1, 2b\,x^2\right)}{8} - \frac{e^{2a}\,\mathrm{Ei}\left(1, -2b\,x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2/x,x)
[Out] -1/2*ln(x)-1/8*exp(-2*a)*Ei(1,2*b*x^2)-1/8*exp(2*a)*Ei(1,-2*b*x^2)
```

maxima [A] time = 0.41, size = 31, normalized size = 0.84

$$\frac{1}{8}\,\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)} + \frac{1}{8}\,\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)} - \frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x,x, algorithm="maxima")
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) - 1/2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sinh\left(b\,x^2 + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^2/x,x)
[Out] int(sinh(a + b*x^2)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2\left(a + bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2/x,x)
[Out] Integral(sinh(a + b*x**2)**2/x, x)
```

3.13 $\int \frac{\sinh^2(a+bx^2)}{x^2} \, dx$

Optimal. Leaf size=88

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}\, e^{-2a}\sqrt{b}\, \text{erf}\left(\sqrt{2}\sqrt{b}\, x\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\, e^{2a}\sqrt{b}\, \text{erfi}\left(\sqrt{2}\sqrt{b}\, x\right)-\frac{\sinh^2\left(a+bx^2\right)}{x}$$

```
[Out] -sinh(b*x^2+a)^2/x-1/4*erf(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/exp(
2*a)+1/4*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5330, 5617, 5314, 5298, 2204, 2205}

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}\, e^{-2a}\sqrt{b}\, \text{Erf}\left(\sqrt{2}\sqrt{b}\, x\right)+\frac{1}{2}\sqrt{\frac{\pi}{2}}\, e^{2a}\sqrt{b}\, \text{Erfi}\left(\sqrt{2}\sqrt{b}\, x\right)-\frac{\sinh^2\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2/x^2,x]
```

```
[Out] -(Sqrt[b]*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(2*E^(2*a)) + (Sqrt[b]*E^(2*a)
*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/2 - Sinh[a + b*x^2]^2/x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5314

```
Int[((a_.) + (b_.)*Sinh[u_])^(p_.), x_Symbol] :> Int[(a + b*Sinh[ExpandToSu
m[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatc
hQ[u, x]
```

Rule 5330

```
Int[(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> -Simp[Sinh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[(b*n*p)/(n - 1), Int[Sinh[a + b
*x^n]^(p - 1)*Cosh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5617

```
Int[Cosh[w_]^(p_.)*(u_.)*Sinh[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin
h[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sinh^2\left(a+bx^2\right)}{x}+(4b)\int \cosh\left(a+bx^2\right)\sinh\left(a+bx^2\right)\,dx\\ &= -\frac{\sinh^2\left(a+bx^2\right)}{x}+(2b)\int \sinh\left(2\left(a+bx^2\right)\right)\,dx\\ &= -\frac{\sinh^2\left(a+bx^2\right)}{x}+(2b)\int \sinh\left(2a+2bx^2\right)\,dx\\ &= -\frac{\sinh^2\left(a+bx^2\right)}{x}-b\int e^{-2a-2bx^2}\,dx+b\int e^{2a+2bx^2}\,dx\\ &= -\frac{1}{2}\sqrt{b}\,e^{-2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{b}\,x\right)+\frac{1}{2}\sqrt{b}\,e^{2a}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)-\frac{\sinh^2\left(a+bx^2\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.22, size = 94, normalized size = 1.07

$$\frac{\sqrt{2\pi}\sqrt{b}\,x(\sinh(2a)-\cosh(2a))\mathrm{erf}\left(\sqrt{2}\sqrt{b}\,x\right)+\sqrt{2\pi}\sqrt{b}\,x(\sinh(2a)+\cosh(2a))\mathrm{erfi}\left(\sqrt{2}\sqrt{b}\,x\right)-4\sinh^2\left(a+bx\right.}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2/x^2,x]

[Out] (Sqrt[b]*Sqrt[2*Pi]*x*Erf[Sqrt[2]*Sqrt[b]*x]*(-Cosh[2*a] + Sinh[2*a]) + Sqr
t[b]*Sqrt[2*Pi]*x*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) - 4*Sinh[
a + b*x^2]^2)/(4*x)
```

fricas [B] time = 0.50, size = 396, normalized size = 4.50

$$-\frac{\cosh\left(bx^2+a\right)^4+4\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^3+\sinh\left(bx^2+a\right)^4+\sqrt{2}\,\sqrt{\pi}\left(x\cosh\left(bx^2+a\right)^2\cosh\left(2\,a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^4 + 4*cosh(b*x^2 + a)*sinh(b*x^2 + a)^3 + sinh(b*x^2
+ a)^4 + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a) + x*cosh(b*x^2 + a
)^2*sinh(2*a) + (x*cosh(2*a) + x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cosh(b
*x^2 + a)*cosh(2*a) + x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b
)*erf(sqrt(2)*sqrt(-b)*x) + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a)
 - x*cosh(b*x^2 + a)^2*sinh(2*a) + (x*cosh(2*a) - x*sinh(2*a))*sinh(b*x^2 +
 a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) - x*cosh(b*x^2 + a)*sinh(2*a))*sinh(
b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) + 2*(3*cosh(b*x^2 + a)^2 - 1)*si
nh(b*x^2 + a)^2 - 2*cosh(b*x^2 + a)^2 + 4*(cosh(b*x^2 + a)^3 - cosh(b*x^2 +
 a))*sinh(b*x^2 + a) + 1)/(x*cosh(b*x^2 + a)^2 + 2*x*cosh(b*x^2 + a)*sinh(b
*x^2 + a) + x*sinh(b*x^2 + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx^2+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^2 + a)^2/x^2, x)
```

maple [A] time = 0.07, size = 86, normalized size = 0.98

$$\frac{1}{2x}-\frac{e^{-2a}e^{-2b\,x^2}}{4x}-\frac{e^{-2a}\sqrt{b}\,\sqrt{\pi}\,\sqrt{2}\,\operatorname{erf}\left(x\sqrt{2}\,\sqrt{b}\right)}{4}-\frac{e^{2a}e^{2b\,x^2}}{4x}+\frac{e^{2a}b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-2b}\,x\right)}{2\sqrt{-2b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2/x^2,x)
```

```
[Out] 1/2/x-1/4*exp(-2*a)/x*exp(-2*b*x^2)-1/4*exp(-2*a)*b^(1/2)*Pi^(1/2)*2^(1/2)*
erf(x*2^(1/2)*b^(1/2))-1/4*exp(2*a)/x*exp(2*b*x^2)+1/2*exp(2*a)*b*Pi^(1/2)/
(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)
```

maxima [A] time = 0.38, size = 61, normalized size = 0.69

$$-\frac{\sqrt{2}\sqrt{bx^2}e^{(-2a)}\Gamma\left(-\frac{1}{2},2bx^2\right)}{8x}-\frac{\sqrt{2}\sqrt{-bx^2}e^{(2a)}\Gamma\left(-\frac{1}{2},-2bx^2\right)}{8x}+\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(b*x^2)*e^(-2*a)*gamma(-1/2, 2*b*x^2)/x - 1/8*sqrt(2)*sqrt
(-b*x^2)*e^(2*a)*gamma(-1/2, -2*b*x^2)/x + 1/2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(b\,x^2+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^2/x^2,x)
```

```
[Out] int(sinh(a + b*x^2)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2/x**2, x)
```

3.14 $\int \frac{\sinh^2(a+bx^2)}{x^3} dx$

Optimal. Leaf size=57

$$\frac{1}{2}b\sinh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{4x^2}$$

```
[Out] 1/4/x^2-1/4*cosh(2*b*x^2+2*a)/x^2+1/2*b*cosh(2*a)*Shi(2*b*x^2)+1/2*b*Chi(2*
b*x^2)*sinh(2*a)
```

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5340, 5321, 3297, 3303, 3298, 3301}

$$\frac{1}{2}b\sinh(2a)\text{Chi}\left(2bx^2\right)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^2/x^3,x]
```

```
[Out] 1/(4*x^2) - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2*a
])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5321

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2\left(a+bx^2\right)}{x^3}\,dx &= \int \left(-\frac{1}{2x^3}+\frac{\cosh\left(2a+2bx^2\right)}{2x^3}\right)dx \\
&= \frac{1}{4x^2}+\frac{1}{2}\int \frac{\cosh\left(2a+2bx^2\right)}{x^3}\,dx \\
&= \frac{1}{4x^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^2\right) \\
&= \frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}b\,\text{Subst}\left(\int \frac{\sinh(2a+2bx)}{x}\,dx,x,x^2\right) \\
&= \frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}(b\cosh(2a))\,\text{Subst}\left(\int \frac{\sinh(2bx)}{x}\,dx,x,x^2\right)+\frac{1}{2}(b\sinh(2a \\
&= \frac{1}{4x^2}-\frac{\cosh\left(2\left(a+bx^2\right)\right)}{4x^2}+\frac{1}{2}b\text{Chi}\left(2bx^2\right)\sinh(2a)+\frac{1}{2}b\cosh(2a)\text{Shi}\left(2bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.81

$$\frac{1}{2}\left(b\sinh(2a)\text{Chi}\left(2bx^2\right)+b\cosh(2a)\text{Shi}\left(2bx^2\right)-\frac{\sinh^2\left(a+bx^2\right)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^2/x^3,x]
```

```
[Out] (b*CoshIntegral[2*b*x^2]*Sinh[2*a] - Sinh[a + b*x^2]^2/x^2 + b*Cosh[2*a]*Si
nhIntegral[2*b*x^2])/2
```

fricas [A] time = 0.46, size = 90, normalized size = 1.58

$$-\frac{\cosh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}\left(2\,bx^2\right)-bx^2\mathrm{Ei}\left(-2\,bx^2\right)\right)\cosh\left(2\,a\right)+\sinh\left(bx^2+a\right)^2-\left(bx^2\mathrm{Ei}\left(2\,bx^2\right)+bx^2\mathrm{Ei}\left(-2\right.\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(cosh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) - b*x^2*Ei(-2*b*x^2))*cosh(2*a
) + sinh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) + b*x^2*Ei(-2*b*x^2))*sinh(2*a)
- 1)/x^2
```

giac [B] time = 0.16, size = 126, normalized size = 2.21

$$\frac{2\left(bx^2+a\right)b^2\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)}-2\,ab^2\mathrm{Ei}\left(2\,bx^2\right)e^{(2\,a)}-2\left(bx^2+a\right)b^2\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)}+2\,ab^2\mathrm{Ei}\left(-2\,bx^2\right)e^{(-2\,a)}-b}{8\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)*b^2*Ei(2*b*x^2)*e^(2*a) - 2*a*b^2*Ei(2*b*x^2)*e^(2*a) -
2*(b*x^2 + a)*b^2*Ei(-2*b*x^2)*e^(-2*a) + 2*a*b^2*Ei(-2*b*x^2)*e^(-2*a) - b
^2*e^(2*b*x^2 + 2*a) - b^2*e^(-2*b*x^2 - 2*a) + 2*b^2)/(b^2*x^2)
```

maple [A] time = 0.06, size = 69, normalized size = 1.21

$$\frac{1}{4x^2}-\frac{\mathrm{e}^{-2a}\mathrm{e}^{-2b\,x^2}}{8x^2}+\frac{\mathrm{e}^{-2a}b\,\mathrm{Ei}\left(1,2b\,x^2\right)}{4}-\frac{\mathrm{e}^{2a}\mathrm{e}^{2b\,x^2}}{8x^2}-\frac{\mathrm{e}^{2a}b\,\mathrm{Ei}\left(1,-2b\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^2/x^3,x)
```

```
[Out] 1/4/x^2-1/8*exp(-2*a)/x^2*exp(-2*b*x^2)+1/4*exp(-2*a)*b*Ei(1,2*b*x^2)-1/8*e
xp(2*a)/x^2*exp(2*b*x^2)-1/4*exp(2*a)*b*Ei(1,-2*b*x^2)
```

maxima [A] time = 0.38, size = 36, normalized size = 0.63

$$-\frac{1}{4}\,be^{(-2\,a)}\Gamma\left(-1,2\,bx^2\right)+\frac{1}{4}\,be^{(2\,a)}\Gamma\left(-1,-2\,bx^2\right)+\frac{1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e^(-2*a)*gamma(-1, 2*b*x^2) + 1/4*b*e^(2*a)*gamma(-1, -2*b*x^2) + 1/
4/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(b\,x^2+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^2/x^3,x)
```

```
[Out] int(sinh(a + b*x^2)^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**2/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**2/x**3, x)
```

3.15 $\int x^3 \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=79

$$-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}+\frac{\sinh\left(a+bx^2\right)}{3b^2}-\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{x^2\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{6b}$$

```
[Out] -1/3*x^2*cosh(b*x^2+a)/b+1/3*sinh(b*x^2+a)/b^2+1/6*x^2*cosh(b*x^2+a)*sinh(b
*x^2+a)^2/b-1/18*sinh(b*x^2+a)^3/b^2
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5320, 3310, 3296, 2637}

$$-\frac{\sinh^3\left(a+bx^2\right)}{18b^2}+\frac{\sinh\left(a+bx^2\right)}{3b^2}-\frac{x^2\cosh\left(a+bx^2\right)}{3b}+\frac{x^2\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^2]^3,x]
```

```
[Out] -(x^2*Cosh[a + b*x^2])/(3*b) + Sinh[a + b*x^2]/(3*b^2) + (x^2*Cosh[a + b*x^
2]*Sinh[a + b*x^2]^2)/(6*b) - Sinh[a + b*x^2]^3/(18*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sinh^3\left(a+bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x\sinh^3(a+bx)\,dx, x, x^2\right)$$
$$= \frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b} - \frac{\sinh^3\left(a+bx^2\right)}{18b^2} - \frac{1}{3}\text{Subst}\left(\int x\sinh(a+bx)\,dx,\right.$$
$$= -\frac{x^2\cosh\left(a+bx^2\right)}{3b} + \frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b} - \frac{\sinh^3\left(a+bx^2\right)}{18b^2} + \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{x^2\cosh\left(a+bx^2\right)}{3b} + \frac{\sinh\left(a+bx^2\right)}{3b^2} + \frac{x^2\cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)}{6b} - \frac{\sinh^3\left(a\right.}{18b}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 0.73

$$-\frac{-27\sinh\left(a+bx^2\right)+\sinh\left(3\left(a+bx^2\right)\right)+27bx^2\cosh\left(a+bx^2\right)-3bx^2\cosh\left(3\left(a+bx^2\right)\right)}{72b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^2]^3,x]
```

```
[Out] -1/72*(27*b*x^2*Cosh[a + b*x^2] - 3*b*x^2*Cosh[3*(a + b*x^2)] - 27*Sinh[a +
 b*x^2] + Sinh[3*(a + b*x^2)])/b^2
```

fricas [A] time = 0.46, size = 94, normalized size = 1.19

$$\frac{3\,bx^2\cosh\left(bx^2+a\right)^3+9\,bx^2\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-27\,bx^2\cosh\left(bx^2+a\right)-\sinh\left(bx^2+a\right)^3-3\left(\cosh\right.}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(3*b*x^2*cosh(b*x^2 + a)^3 + 9*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2
 - 27*b*x^2*cosh(b*x^2 + a) - sinh(b*x^2 + a)^3 - 3*(cosh(b*x^2 + a)^2 - 9)
*sinh(b*x^2 + a))/b^2
```

giac [B] time = 0.21, size = 183, normalized size = 2.32

$$\frac{3\left(bx^2+a\right)e^{\left(3\,bx^2+3\,a\right)}-3\,ae^{\left(3\,bx^2+3\,a\right)}-27\left(bx^2+a\right)e^{\left(bx^2+a\right)}+27\,ae^{\left(bx^2+a\right)}-27\left(bx^2+a\right)e^{\left(-bx^2-a\right)}+27\,ae^{\left(-bx^2-a\right)}}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/144*(3*(b*x^2 + a)*e^(3*b*x^2 + 3*a) - 3*a*e^(3*b*x^2 + 3*a) - 27*(b*x^2
+ a)*e^(b*x^2 + a) + 27*a*e^(b*x^2 + a) - 27*(b*x^2 + a)*e^(-b*x^2 - a) + 2
7*a*e^(-b*x^2 - a) + 3*(b*x^2 + a)*e^(-3*b*x^2 - 3*a) - 3*a*e^(-3*b*x^2 - 3
*a) - e^(3*b*x^2 + 3*a) + 27*e^(b*x^2 + a) - 27*e^(-b*x^2 - a) + e^(-3*b*x^
2 - 3*a))/b^2
```

maple [A] time = 0.09, size = 93, normalized size = 1.18

$$\frac{\left(3b\,x^2-1\right)\mathrm{e}^{3b\,x^2+3a}}{144b^2}-\frac{3\left(b\,x^2-1\right)\mathrm{e}^{b\,x^2+a}}{16b^2}-\frac{3\left(b\,x^2+1\right)\mathrm{e}^{-b\,x^2-a}}{16b^2}+\frac{\left(3b\,x^2+1\right)\mathrm{e}^{-3b\,x^2-3a}}{144b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^2+a)^3,x)
```

```
[Out] 1/144*(3*b*x^2-1)/b^2*exp(3*b*x^2+3*a)-3/16*(b*x^2-1)/b^2*exp(b*x^2+a)-3/16
*(b*x^2+1)/b^2*exp(-b*x^2-a)+1/144*(3*b*x^2+1)/b^2*exp(-3*b*x^2-3*a)
```

maxima [A] time = 0.35, size = 100, normalized size = 1.27

$$\frac{\left(3\,bx^2e^{(3\,a)}-e^{(3\,a)}\right)e^{\left(3\,bx^2\right)}}{144\,b^2}-\frac{3\left(bx^2e^a-e^a\right)e^{\left(bx^2\right)}}{16\,b^2}-\frac{3\left(bx^2+1\right)e^{\left(-bx^2-a\right)}}{16\,b^2}+\frac{\left(3\,bx^2+1\right)e^{\left(-3\,bx^2-3\,a\right)}}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/144*(3*b*x^2*e^(3*a) - e^(3*a))*e^(3*b*x^2)/b^2 - 3/16*(b*x^2*e^a - e^a)*
e^(b*x^2)/b^2 - 3/16*(b*x^2 + 1)*e^(-b*x^2 - a)/b^2 + 1/144*(3*b*x^2 + 1)*e
^(-3*b*x^2 - 3*a)/b^2
```

mupad [B] time = 0.13, size = 70, normalized size = 0.89

$$\frac{\frac{x^2\cosh\left(b\,x^2+a\right)^3}{6}-\frac{x^2\cosh\left(b\,x^2+a\right)}{2}}{b}+\frac{7\sinh\left(b\,x^2+a\right)}{18\,b^2}-\frac{\cosh\left(b\,x^2+a\right)^2\sinh\left(b\,x^2+a\right)}{18\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a + b*x^2)^3,x)
```

```
[Out] ((x^2*cosh(a + b*x^2)^3)/6 - (x^2*cosh(a + b*x^2))/2)/b + (7*sinh(a + b*x^2
))/(18*b^2) - (cosh(a + b*x^2)^2*sinh(a + b*x^2))/(18*b^2)
```

sympy [A] time = 2.77, size = 92, normalized size = 1.16

$$\begin{cases} \frac{x^2 \sinh^2\left(a+bx^2\right) \cosh\left(a+bx^2\right)}{2b} - \frac{x^2 \cosh^3\left(a+bx^2\right)}{3b} - \frac{7 \sinh^3\left(a+bx^2\right)}{18b^2} + \frac{\sinh\left(a+bx^2\right) \cosh^2\left(a+bx^2\right)}{3b^2} & \text{for } b \neq 0 \\ \frac{x^4 \sinh^3(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**2+a)**3,x)
```

```
[Out] Piecewise((x**2*sinh(a + b*x**2)**2*cosh(a + b*x**2)/(2*b) - x**2*cosh(a +
b*x**2)**3/(3*b) - 7*sinh(a + b*x**2)**3/(18*b**2) + sinh(a + b*x**2)*cosh(
a + b*x**2)**2/(3*b**2), Ne(b, 0)), (x**4*sinh(a)**3/4, True))
```

3.16 $\int x^2 \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=160

$$\frac{3\sqrt{\pi}\,e^{-a}\mathrm{erf}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{-3a}\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}+\frac{3\sqrt{\pi}\,e^{a}\mathrm{erfi}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{3a}\mathrm{erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}-\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\,\mathrm{co}}{}$$

```
[Out] -3/8*x*cosh(b*x^2+a)/b+1/24*x*cosh(3*b*x^2+3*a)/b-1/288*erf(x*3^(1/2)*b^(1/
2))*3^(1/2)*Pi^(1/2)/b^(3/2)/exp(3*a)-1/288*exp(3*a)*erfi(x*3^(1/2)*b^(1/2)
)*3^(1/2)*Pi^(1/2)/b^(3/2)+3/32*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)+3/32
*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5340, 5324, 5299, 2204, 2205}

$$\frac{3\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{-3a}\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}+\frac{3\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{3a}\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{96b^{3/2}}-\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-3*x*Cosh[a + b*x^2])/(8*b) + (x*Cosh[3*a + 3*b*x^2])/(24*b) + (3*Sqrt[Pi]
*Erf[Sqrt[b]*x])/(32*b^(3/2)*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[b]*x])/(96
*b^(3/2)*E^(3*a)) + (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(32*b^(3/2)) - (E^(3*a
)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \sinh^3\left(a+bx^2\right) dx &= \int \left(-\frac{3}{4}x^2 \sinh\left(a+bx^2\right)+\frac{1}{4}x^2\sinh\left(3a+3bx^2\right)\right) dx\\
&= \frac{1}{4}\int x^2\sinh\left(3a+3bx^2\right) dx - \frac{3}{4}\int x^2 \sinh\left(a+bx^2\right) dx\\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}-\frac{\int \cosh\left(3a+3bx^2\right) dx}{24b}+\frac{3\int \cosh\left(a+
\\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}-\frac{\int e^{-3a-3bx^2} dx}{48b}-\frac{\int e^{3a+3bx^2} dx}{48b}+\frac{3\int e^{-a-}}{16
\\
&= -\frac{3x\cosh\left(a+bx^2\right)}{8b}+\frac{x\cosh\left(3a+3bx^2\right)}{24b}+\frac{3e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{32b^{3/2}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\right.}{96b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.30, size = 184, normalized size = 1.15

$$\frac{27\sqrt{\pi}\,(\cosh(a)-\sinh(a))\mathrm{erf}\left(\sqrt{b}\,x\right)+\sqrt{3\pi}\,(\sinh(3a)-\cosh(3a))\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)+27\sqrt{\pi}\,\sinh(a)\mathrm{erfi}\left(\sqrt{b}\,x\right)-\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-108*Sqrt[b]*x*Cosh[a + b*x^2] + 12*Sqrt[b]*x*Cosh[3*(a + b*x^2)] + 27*Sqr
t[Pi]*Cosh[a]*Erfi[Sqrt[b]*x] - Sqrt[3*Pi]*Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x
] + 27*Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + 27*Sqrt[Pi]*Erfi[Sqrt[
```

```
b]*x]*Sinh[a] - Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Sqrt[3*Pi]*E
rf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] + Sinh[3*a]))/(288*b^(3/2))
```

fricas [B] time = 0.43, size = 904, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/288*(6*b*x*cosh(b*x^2 + a)^6 + 36*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 +
 6*b*x*sinh(b*x^2 + a)^6 - 54*b*x*cosh(b*x^2 + a)^4 + 18*(5*b*x*cosh(b*x^2
+ a)^2 - 3*b*x)*sinh(b*x^2 + a)^4 - 54*b*x*cosh(b*x^2 + a)^2 + 24*(5*b*x*co
sh(b*x^2 + a)^3 - 9*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)*sqrt(p
i)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) + sinh(3*a))*sinh(b*x^2 + a)^3
 + cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) + cosh(b*x^2
+ a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) + cosh(b
*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(3)*sqrt(-b)*x) -
sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) - sinh(3*a))*sin
h(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a)
 - cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh
(3*a) - cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*s
qrt(b)*x) - 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) + sinh(a))*si
nh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) +
cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) +
 cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 27*
sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a)^3
 - cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) - cosh(b*x^2 + a)
*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) - cosh(b*x^2 + a
)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 18*(5*b*x*cosh(b*x^2
 + a)^4 - 18*b*x*cosh(b*x^2 + a)^2 - 3*b*x)*sinh(b*x^2 + a)^2 + 6*b*x + 36*
(b*x*cosh(b*x^2 + a)^5 - 6*b*x*cosh(b*x^2 + a)^3 - 3*b*x*cosh(b*x^2 + a))*s
inh(b*x^2 + a))/(b^2*cosh(b*x^2 + a)^3 + 3*b^2*cosh(b*x^2 + a)^2*sinh(b*x^2
 + a) + 3*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 + b^2*sinh(b*x^2 + a)^3)
```

giac [A] time = 0.20, size = 166, normalized size = 1.04

$$\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{-b}\,x\right)e^{(3\,a)}}{288\,\sqrt{-b}\,b}+\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{b}\,x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{\left(3\,bx^2+3\,a\right)}}{48\,b}-\frac{3\,xe^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,xe^{\left(-bx^2-a\right)}}{16\,b}+\frac{xe^{\left(-3\,bx^2-3\right.}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/288*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) + 1/28
8*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b
```

```
*x^2 + 3*a)/b - 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b + 1/48*x*e
^(-3*b*x^2 - 3*a)/b - 3/32*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/b^(3/2) - 3/32*s
qrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

maple [A] time = 0.12, size = 157, normalized size = 0.98

$$\frac{e^{-3a} x\, e^{-3b x^2}}{48b}-\frac{e^{-3a}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{288b^{\frac{3}{2}}}-\frac{3\,e^{-a} x\, e^{-b x^2}}{16b}+\frac{3\,e^{-a}\sqrt{\pi}\,\operatorname{erf}\left(x\sqrt{b}\right)}{32b^{\frac{3}{2}}}+\frac{e^{3a} x\, e^{3b x^2}}{48b}-\frac{e^{3a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-3b}\,x\right)}{96b\sqrt{-3b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(b*x^2+a)^3,x)
```

```
[Out] 1/48*exp(-3*a)/b*x*exp(-3*b*x^2)-1/288*exp(-3*a)/b^(3/2)*Pi^(1/2)*3^(1/2)*e
rf(x*3^(1/2)*b^(1/2))-3/16*exp(-a)/b*x*exp(-b*x^2)+3/32*exp(-a)/b^(3/2)*Pi^
(1/2)*erf(x*b^(1/2))+1/48*exp(3*a)/b*x*exp(3*b*x^2)-1/96*exp(3*a)/b*Pi^(1/2
)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)-3/16*exp(a)*exp(b*x^2)*x/b+3/32*exp(a)/b
*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

maxima [A] time = 0.43, size = 162, normalized size = 1.01

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{-b}\,x\right)e^{(3\,a)}}{288\,\sqrt{-b}\,b}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)e^{(-3\,a)}}{288\,b^{\frac{3}{2}}}+\frac{xe^{\left(3\,bx^2+3\,a\right)}}{48\,b}-\frac{3\,xe^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,xe^{\left(-bx^2-a\right)}}{16\,b}+\frac{xe^{\left(-3\,bx^2-3\,a\right)}}{48\,b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/(sqrt(-b)*b) - 1/28
8*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/b^(3/2) + 1/48*x*e^(3*b*
x^2 + 3*a)/b - 3/16*x*e^(b*x^2 + a)/b - 3/16*x*e^(-b*x^2 - a)/b + 1/48*x*e^
(-3*b*x^2 - 3*a)/b + 3/32*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(3/2) + 3/32*sqr
t(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sinh\left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*x^2)^3,x)
```

```
[Out] int(x^2*sinh(a + b*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh^3\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(b*x**2+a)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**2)**3, x)
```

3.17 $\int x\sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{\cosh^3\left(a+bx^2\right)}{6b}-\frac{\cosh\left(a+bx^2\right)}{2b}$$

```
[Out] -1/2*cosh(b*x^2+a)/b+1/6*cosh(b*x^2+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2633}

$$\frac{\cosh^3\left(a+bx^2\right)}{6b}-\frac{\cosh\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2]^3,x]
```

```
[Out] -Cosh[a + b*x^2]/(2*b) + Cosh[a + b*x^2]^3/(6*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\sinh^3\left(a+bx^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \sinh^3(a+bx)\,dx,x,x^2\right)\\
&= -\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh\left(a+bx^2\right)\right)}{2b}\\
&= -\frac{\cosh\left(a+bx^2\right)}{2b}+\frac{\cosh^3\left(a+bx^2\right)}{6b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{\cosh\left(3\left(a+bx^2\right)\right)}{24b}-\frac{3\cosh\left(a+bx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2]^3,x]
```

```
[Out] (-3*Cosh[a + b*x^2])/(8*b) + Cosh[3*(a + b*x^2)]/(24*b)
```

fricas [A] time = 0.52, size = 46, normalized size = 1.39

$$\frac{\cosh\left(bx^2+a\right)^3+3\,\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^2-9\,\cosh\left(bx^2+a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 - 9*cosh(b*x^
2 + a))/b
```

giac [A] time = 0.24, size = 56, normalized size = 1.70

$$-\frac{\left(9\,e^{\left(2\,bx^2+2\,a\right)}-1\right)e^{\left(-3\,bx^2-3\,a\right)}-e^{\left(3\,bx^2+3\,a\right)}+9\,e^{\left(bx^2+a\right)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*((9*e^(2*b*x^2 + 2*a) - 1)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2 + 3*a) + 9
*e^(b*x^2 + a))/b
```

maple [A] time = 0.02, size = 28, normalized size = 0.85

$$\frac{\left(-\frac{2}{3}+\frac{\left(\sinh^2\left(b\,x^2+a\right)\right)}{3}\right)\cosh\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a)^3,x)
```

```
[Out] 1/2/b*(-2/3+1/3*sinh(b*x^2+a)^2)*cosh(b*x^2+a)
```

maxima [B] time = 0.32, size = 62, normalized size = 1.88

$$\frac{e^{\left(3\,bx^2+3\,a\right)}}{48\,b}-\frac{3\,e^{\left(bx^2+a\right)}}{16\,b}-\frac{3\,e^{\left(-bx^2-a\right)}}{16\,b}+\frac{e^{\left(-3\,bx^2-3\,a\right)}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*e^(3*b*x^2 + 3*a)/b - 3/16*e^(b*x^2 + a)/b - 3/16*e^(-b*x^2 - a)/b + 1
/48*e^(-3*b*x^2 - 3*a)/b
```

mupad [B] time = 0.06, size = 28, normalized size = 0.85

$$-\frac{3\cosh\left(b\,x^2+a\right)-\cosh\left(b\,x^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^2)^3,x)
```

```
[Out] -(3*cosh(a + b*x^2) - cosh(a + b*x^2)^3)/(6*b)
```

sympy [A] time = 0.78, size = 44, normalized size = 1.33

$$\begin{cases} \frac{\sinh^2\left(a+bx^2\right)\cosh\left(a+bx^2\right)}{2b}-\frac{\cosh^3\left(a+bx^2\right)}{3b} & \text{for } b\neq 0 \\ \frac{x^2\sinh^3(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x**2)**2*cosh(a + b*x**2)/(2*b) - cosh(a + b*x**2)**3
/(3*b), Ne(b, 0)), (x**2*sinh(a)**3/2, True))
```

3.18 $\int \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=125

$$\frac{3\sqrt{\pi}\,e^{-a}\mathrm{erf}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{-3a}\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}-\frac{3\sqrt{\pi}\,e^{a}\mathrm{erfi}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{3a}\mathrm{erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}$$

```
[Out] -1/48*erf(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a)/b^(1/2)+1/48*exp(3*a
)*erfi(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)+3/16*erf(x*b^(1/2))*Pi^(
1/2)/exp(a)/b^(1/2)-3/16*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5300, 5298, 2204, 2205}

$$\frac{3\sqrt{\pi}\,e^{-a}\mathrm{Erf}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}-\frac{\sqrt{\frac{\pi}{3}}\,e^{-3a}\mathrm{Erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}-\frac{3\sqrt{\pi}\,e^{a}\mathrm{Erfi}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{3a}\mathrm{Erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi]*Erf[Sqrt[b]*x])/(16*Sqrt[b]*E^a) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt
[b]*x])/(16*Sqrt[b]*E^(3*a)) - (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/(16*Sqrt[b]
) + (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(16*Sqrt[b])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5300

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3\left(a+bx^2\right)\,dx &= \int \left(-\frac{3}{4}\sinh\left(a+bx^2\right)+\frac{1}{4}\sinh\left(3a+3bx^2\right)\right)dx \\
&= \frac{1}{4}\int \sinh\left(3a+3bx^2\right)\,dx-\frac{3}{4}\int \sinh\left(a+bx^2\right)\,dx \\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^2}\,dx\right)+\frac{1}{8}\int e^{3a+3bx^2}\,dx+\frac{3}{8}\int e^{-a-bx^2}\,dx-\frac{3}{8}\int e^{a+bx^2}\,dx \\
&= \frac{3e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}-\frac{e^{-3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)}{16\sqrt{b}}-\frac{3e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\right)}{16\sqrt{b}}+\frac{e^{3a}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\sqrt{3}\,\sqrt{b}\,x\right.}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 136, normalized size = 1.09

$$\frac{\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\,(\cosh(a)-\sinh(a))\mathrm{erf}\left(\sqrt{b}\,x\right)+(\sinh(3a)-\cosh(3a))\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)-3\sqrt{3}\,\sinh(a)\mathrm{erfi}\left(\sqrt{b}\,x\right)+\sinh\right.}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[a]*Erfi[Sqrt[b]*x] + Cosh[3*a]*Erfi[Sqrt[3]*Sq
rt[b]*x] + 3*Sqrt[3]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - 3*Sqrt[3]*Erfi[Sq
rt[b]*x]*Sinh[a] + Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Erf[Sqrt[3]*Sqrt[b]*
x]*(-Cosh[3*a] + Sinh[3*a])))/(16*Sqrt[b])
```

fricas [A] time = 0.44, size = 112, normalized size = 0.90

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\sqrt{-b}\,(\cosh(3\,a)+\sinh(3\,a))\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{-b}\,x\right)+\sqrt{3}\,\sqrt{\pi}\,\sqrt{b}\,(\cosh(3\,a)-\sinh(3\,a))\,\mathrm{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)-9\,\sqrt{\pi}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-b)*(cosh(3*a) + sinh(3*a))*erf(sqrt(3)*sqrt(-
b)*x) + sqrt(3)*sqrt(pi)*sqrt(b)*(cosh(3*a) - sinh(3*a))*erf(sqrt(3)*sqrt(b
```

```
)*x) - 9*sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - 9*sqrt(pi)
*sqrt(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b
```

giac [A] time = 0.31, size = 95, normalized size = 0.76

$$-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{-b}\,x\right)e^{(3\,a)}}{48\,\sqrt{-b}}+\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{3}\,\sqrt{b}\,x\right)e^{(-3\,a)}}{48\,\sqrt{b}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,x\right)e^{(-a)}}{16\,\sqrt{b}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-b}\,x\right)e^{a}}{16\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) + 1/48*sqr
t(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) - 3/16*sqrt(pi)*erf(
-sqrt(b)*x)*e^(-a)/sqrt(b) + 3/16*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)
```

maple [A] time = 0.06, size = 86, normalized size = 0.69

$$-\frac{\mathrm{e}^{-3a}\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{48\sqrt{b}}+\frac{3\operatorname{erf}\left(x\sqrt{b}\right)\sqrt{\pi}\,\mathrm{e}^{-a}}{16\sqrt{b}}+\frac{\mathrm{e}^{3a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-3b}\,x\right)}{16\sqrt{-3b}}-\frac{3\,\mathrm{e}^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)}{16\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3,x)
```

```
[Out] -1/48*exp(-3*a)*Pi^(1/2)*3^(1/2)/b^(1/2)*erf(x*3^(1/2)*b^(1/2))+3/16*erf(x*
b^(1/2))*Pi^(1/2)*exp(-a)/b^(1/2)+1/16*exp(3*a)*Pi^(1/2)/(-3*b)^(1/2)*erf((
-3*b)^(1/2)*x)-3/16*exp(a)*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

maxima [A] time = 0.40, size = 91, normalized size = 0.73

$$\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{-b}\,x\right)e^{(3\,a)}}{48\,\sqrt{-b}}-\frac{\sqrt{3}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{3}\,\sqrt{b}\,x\right)e^{(-3\,a)}}{48\,\sqrt{b}}+\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{b}\,x\right)e^{(-a)}}{16\,\sqrt{b}}-\frac{3\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b}\,x\right)e^{a}}{16\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) - 1/48*sqrt(
3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqr
t(b)*x)*e^(-a)/sqrt(b) - 3/16*sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^3,x)
```

```
[Out] int(sinh(a + b*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^3\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3, x)
```

3.19 $\int \frac{\sinh^3(a+bx^2)}{x}\,dx$

Optimal. Leaf size=55

$$-\frac{3}{8}\sinh(a)\text{Chi}\left(bx^2\right)+\frac{1}{8}\sinh(3a)\text{Chi}\left(3bx^2\right)-\frac{3}{8}\cosh(a)\text{Shi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\text{Shi}\left(3bx^2\right)$$

```
[Out] -3/8*cosh(a)*Shi(b*x^2)+1/8*cosh(3*a)*Shi(3*b*x^2)-3/8*Chi(b*x^2)*sinh(a)+1
/8*Chi(3*b*x^2)*sinh(3*a)
```

Rubi [A] time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5340, 5318, 5317, 5316}

$$-\frac{3}{8}\sinh(a)\text{Chi}\left(bx^2\right)+\frac{1}{8}\sinh(3a)\text{Chi}\left(3bx^2\right)-\frac{3}{8}\cosh(a)\text{Shi}\left(bx^2\right)+\frac{1}{8}\cosh(3a)\text{Shi}\left(3bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a])/8 + (CoshIntegral[3*b*x^2]*Sinh[3*a])/8 -
(3*Cosh[a]*SinhIntegral[b*x^2])/8 + (Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x}\,dx = \int \left(-\frac{3\sinh\left(a+bx^2\right)}{4x}+\frac{\sinh\left(3a+3bx^2\right)}{4x}\right)dx$$
$$= \frac{1}{4}\int \frac{\sinh\left(3a+3bx^2\right)}{x}\,dx - \frac{3}{4}\int \frac{\sinh\left(a+bx^2\right)}{x}\,dx$$
$$= -\left(\frac{1}{4}(3\cosh(a))\int \frac{\sinh\left(bx^2\right)}{x}\,dx\right) + \frac{1}{4}\cosh(3a)\int \frac{\sinh\left(3bx^2\right)}{x}\,dx - \frac{1}{4}(3\sinh(a))\int \frac{\text{co}}{}$$
$$= -\frac{3}{8}\text{Chi}\left(bx^2\right)\sinh(a) + \frac{1}{8}\text{Chi}\left(3bx^2\right)\sinh(3a) - \frac{3}{8}\cosh(a)\text{Shi}\left(bx^2\right) + \frac{1}{8}\cosh(3a)\text{Shi}\left(3\right.$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.89

$$\frac{1}{8}\left(-3\sinh(a)\text{Chi}\left(bx^2\right)+\sinh(3a)\text{Chi}\left(3bx^2\right)-3\cosh(a)\text{Shi}\left(bx^2\right)+\cosh(3a)\text{Shi}\left(3bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^2]*Sinh[a] + CoshIntegral[3*b*x^2]*Sinh[3*a] - 3*Cosh[
a]*SinhIntegral[b*x^2] + Cosh[3*a]*SinhIntegral[3*b*x^2])/8
```

fricas [A] time = 0.44, size = 83, normalized size = 1.51

$$\frac{1}{16}\left(\text{Ei}\left(3\,bx^2\right)-\text{Ei}\left(-3\,bx^2\right)\right)\cosh\left(3\,a\right)-\frac{3}{16}\left(\text{Ei}\left(bx^2\right)-\text{Ei}\left(-bx^2\right)\right)\cosh(a)+\frac{1}{16}\left(\text{Ei}\left(3\,bx^2\right)+\text{Ei}\left(-3\,bx^2\right)\right)\sinh\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(Ei(3*b*x^2) - Ei(-3*b*x^2))*cosh(3*a) - 3/16*(Ei(b*x^2) - Ei(-b*x^2))
*cosh(a) + 1/16*(Ei(3*b*x^2) + Ei(-3*b*x^2))*sinh(3*a) - 3/16*(Ei(b*x^2) +
Ei(-b*x^2))*sinh(a)
```

giac [A] time = 0.24, size = 50, normalized size = 0.91

$$\frac{1}{16}\,\text{Ei}\left(3\,bx^2\right)e^{(3\,a)}+\frac{3}{16}\,\text{Ei}\left(-bx^2\right)e^{(-a)}-\frac{1}{16}\,\text{Ei}\left(-3\,bx^2\right)e^{(-3\,a)}-\frac{3}{16}\,\text{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) - 1/16*Ei(-3*b*x^2)*e^(-3
*a) - 3/16*Ei(b*x^2)*e^a
```

maple [A] time = 0.08, size = 55, normalized size = 1.00

$$\frac{e^{-3a}\,\mathrm{Ei}\left(1,3b\,x^2\right)}{16}-\frac{3\,e^{-a}\,\mathrm{Ei}\left(1,b\,x^2\right)}{16}-\frac{e^{3a}\,\mathrm{Ei}\left(1,-3b\,x^2\right)}{16}+\frac{3\,e^{a}\,\mathrm{Ei}\left(1,-b\,x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3/x,x)
```

```
[Out] 1/16*exp(-3*a)*Ei(1,3*b*x^2)-3/16*exp(-a)*Ei(1,b*x^2)-1/16*exp(3*a)*Ei(1,-3
*b*x^2)+3/16*exp(a)*Ei(1,-b*x^2)
```

maxima [A] time = 0.43, size = 50, normalized size = 0.91

$$\frac{1}{16}\,\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}+\frac{3}{16}\,\mathrm{Ei}\left(-bx^2\right)e^{(-a)}-\frac{1}{16}\,\mathrm{Ei}\left(-3\,bx^2\right)e^{(-3\,a)}-\frac{3}{16}\,\mathrm{Ei}\left(bx^2\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) - 1/16*Ei(-3*b*x^2)*e^(-3
*a) - 3/16*Ei(b*x^2)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(b\,x^2+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^3/x,x)
```

```
[Out] int(sinh(a + b*x^2)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x, x)
```

3.20 $\int \frac{\sinh^3(a+bx^2)}{x^2} dx$

Optimal. Leaf size=136

$$-\frac{3}{8}\sqrt{\pi}e^{-a}\sqrt{b}\,\text{erf}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{3\pi}e^{-3a}\sqrt{b}\,\text{erf}\left(\sqrt{3}\sqrt{b}\,x\right)-\frac{3}{8}\sqrt{\pi}e^{a}\sqrt{b}\,\text{erfi}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{3\pi}e^{3a}\sqrt{b}\,\text{erfi}\left(\sqrt{3}\sqrt{b}\,x\right)-\frac{\sinh$$

```
[Out] -sinh(b*x^2+a)^3/x-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)-3/8*exp(a)*er
fi(x*b^(1/2))*b^(1/2)*Pi^(1/2)+1/8*erf(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*P
i^(1/2)/exp(3*a)+1/8*exp(3*a)*erfi(x*3^(1/2)*b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1
/2)
```

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5330, 5618, 5299, 2204, 2205}

$$-\frac{3}{8}\sqrt{\pi}e^{-a}\sqrt{b}\,\text{Erf}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{3\pi}e^{-3a}\sqrt{b}\,\text{Erf}\left(\sqrt{3}\sqrt{b}\,x\right)-\frac{3}{8}\sqrt{\pi}e^{a}\sqrt{b}\,\text{Erfi}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{3\pi}e^{3a}\sqrt{b}\,\text{Erfi}\left(\sqrt{3}\sqrt{b}\,x\right)-\frac{\sin$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]*x])/(8*E^a) + (Sqrt[b]*Sqrt[3*Pi]*Erf[Sqrt
[3]*Sqrt[b]*x])/(8*E^(3*a)) - (3*Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqrt[b]*x])/8 +
(Sqrt[b]*E^(3*a)*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[b]*x])/8 - Sinh[a + b*x^2]^3/
x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5330

```
Int[(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_)]^(p_), x_Symbol] :> -Simp[Sinh[
a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[(b*n*p)/(n - 1), Int[Sinh[a + b
*x^n]^(p - 1)*Cosh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p
] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]
```

Rule 5618

```
Int[Cosh[w_]^(q_.)*Sinh[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sinh[v
]^p*Cosh[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] &&
 PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x
]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\sinh^3\left(a+bx^2\right)}{x}+(6b)\int \cosh\left(a+bx^2\right)\sinh^2\left(a+bx^2\right)\,dx\\ &= -\frac{\sinh^3\left(a+bx^2\right)}{x}+(6b)\int \left(-\frac{1}{4}\cosh\left(a+bx^2\right)+\frac{1}{4}\cosh\left(3a+3bx^2\right)\right)dx\\ &= -\frac{\sinh^3\left(a+bx^2\right)}{x}-\frac{1}{2}(3b)\int \cosh\left(a+bx^2\right)\,dx+\frac{1}{2}(3b)\int \cosh\left(3a+3bx^2\right)\,dx\\ &= -\frac{\sinh^3\left(a+bx^2\right)}{x}+\frac{1}{4}(3b)\int e^{-3a-3bx^2}\,dx-\frac{1}{4}(3b)\int e^{-a-bx^2}\,dx-\frac{1}{4}(3b)\int e^{a+bx^2}\,dx+\frac{1}{4}\\ &= -\frac{3}{8}\sqrt{b}\,e^{-a}\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{b}\,e^{-3a}\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{b}\,x\right)-\frac{3}{8}\sqrt{b}\,e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}\,x\right)+\frac{1}{8}\sqrt{}\end{aligned}$$

Mathematica [A] time = 0.31, size = 204, normalized size = 1.50

$$\frac{3\sqrt{\pi}\sqrt{b}\,x(\sinh(a)-\cosh(a))\operatorname{erf}\left(\sqrt{b}\,x\right)+\sqrt{3\pi}\sqrt{b}\,x(\cosh(3a)-\sinh(3a))\operatorname{erf}\left(\sqrt{3}\sqrt{b}\,x\right)-3\sqrt{\pi}\sqrt{b}\,x\sinh(a)\operatorname{er}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3/x^2,x]
```

```
[Out] (-3*Sqrt[b]*Sqrt[Pi]*x*Cosh[a]*Erfi[Sqrt[b]*x] + Sqrt[b]*Sqrt[3*Pi]*x*Cosh[
3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] - 3*Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*Sinh[a]
```

```
 + 3*Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Sqrt[b]*Sqrt[
3*Pi]*x*Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh[3*a]) + Sqrt[b]*Sqrt[3*Pi]
*x*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + 6*Sinh[a + b*x^2] - 2*Sinh[3*(a + b*
x^2)])/(8*x)
```

fricas [B] time = 0.51, size = 892, normalized size = 6.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(b*x^2 + a)^6 + 6*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 + sinh(b*x^2
+ a)^6 + 3*(5*cosh(b*x^2 + a)^2 - 1)*sinh(b*x^2 + a)^4 - 3*cosh(b*x^2 + a)^
4 + 4*(5*cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)
*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) + x*cosh(b*x^2 + a)^3*sinh(3*a) +
(x*cosh(3*a) + x*sinh(3*a))*sinh(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(3
*a) + x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)
^2*cosh(3*a) + x*cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf
(sqrt(3)*sqrt(-b)*x) - sqrt(3)*sqrt(pi)*(x*cosh(b*x^2 + a)^3*cosh(3*a) - x*
cosh(b*x^2 + a)^3*sinh(3*a) + (x*cosh(3*a) - x*sinh(3*a))*sinh(b*x^2 + a)^3
 + 3*(x*cosh(b*x^2 + a)*cosh(3*a) - x*cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2
 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(3*a) - x*cosh(b*x^2 + a)^2*sinh(3*a))
*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*sqrt(b)*x) - 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) + x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) + x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) + x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) + x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 3*sqrt(pi)*(x*cosh(b*x^2
 + a)^3*cosh(a) - x*cosh(b*x^2 + a)^3*sinh(a) + (x*cosh(a) - x*sinh(a))*sin
h(b*x^2 + a)^3 + 3*(x*cosh(b*x^2 + a)*cosh(a) - x*cosh(b*x^2 + a)*sinh(a))*
sinh(b*x^2 + a)^2 + 3*(x*cosh(b*x^2 + a)^2*cosh(a) - x*cosh(b*x^2 + a)^2*si
nh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 3*(5*cosh(b*x^2 + a)^4 - 6
*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^2 + 3*cosh(b*x^2 + a)^2 + 6*(cosh(b
*x^2 + a)^5 - 2*cosh(b*x^2 + a)^3 + cosh(b*x^2 + a))*sinh(b*x^2 + a) - 1)/(
x*cosh(b*x^2 + a)^3 + 3*x*cosh(b*x^2 + a)^2*sinh(b*x^2 + a) + 3*x*cosh(b*x^
2 + a)*sinh(b*x^2 + a)^2 + x*sinh(b*x^2 + a)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(bx^2+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^2 + a)^3/x^2, x)
```

maple [A] time = 0.10, size = 149, normalized size = 1.10

$$\frac{e^{-3a}e^{-3b\,x^2}}{8x}+\frac{e^{-3a}\sqrt{b}\,\sqrt{\pi}\,\sqrt{3}\,\operatorname{erf}\left(x\sqrt{3}\,\sqrt{b}\right)}{8}-\frac{3\,e^{-a}e^{-b\,x^2}}{8x}-\frac{3\,e^{-a}\sqrt{b}\,\sqrt{\pi}\,\operatorname{erf}\left(x\sqrt{b}\right)}{8}-\frac{e^{3a}e^{3b\,x^2}}{8x}+\frac{3\,e^{3a}b\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{\right.}}{8\sqrt{-3b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3/x^2,x)
```

```
[Out] 1/8*exp(-3*a)/x*exp(-3*b*x^2)+1/8*exp(-3*a)*b^(1/2)*Pi^(1/2)*3^(1/2)*erf(x*
3^(1/2)*b^(1/2))-3/8*exp(-a)/x*exp(-b*x^2)-3/8*exp(-a)*b^(1/2)*Pi^(1/2)*erf
(x*b^(1/2))-1/8*exp(3*a)/x*exp(3*b*x^2)+3/8*exp(3*a)*b*Pi^(1/2)/(-3*b)^(1/2
)*erf((-3*b)^(1/2)*x)+3/8*exp(a)*exp(b*x^2)/x-3/8*exp(a)*b*Pi^(1/2)/(-b)^(1
/2)*erf((-b)^(1/2)*x)
```

maxima [A] time = 0.44, size = 102, normalized size = 0.75

$$\frac{\sqrt{3}\,\sqrt{bx^2}\,e^{(-3\,a)}\Gamma\left(-\frac{1}{2},3\,bx^2\right)}{16\,x}-\frac{\sqrt{3}\,\sqrt{-bx^2}\,e^{(3\,a)}\Gamma\left(-\frac{1}{2},-3\,bx^2\right)}{16\,x}-\frac{3\,\sqrt{bx^2}\,e^{(-a)}\Gamma\left(-\frac{1}{2},bx^2\right)}{16\,x}+\frac{3\,\sqrt{-bx^2}\,e^{a}\Gamma\left(-\frac{1}{2},-bx^2\right)}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(3)*sqrt(b*x^2)*e^(-3*a)*gamma(-1/2, 3*b*x^2)/x - 1/16*sqrt(3)*sqr
t(-b*x^2)*e^(3*a)*gamma(-1/2, -3*b*x^2)/x - 3/16*sqrt(b*x^2)*e^(-a)*gamma(-
1/2, b*x^2)/x + 3/16*sqrt(-b*x^2)*e^a*gamma(-1/2, -b*x^2)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(b\,x^2+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^3/x^2,x)
```

```
[Out] int(sinh(a + b*x^2)^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x**2, x)
```

3.21 $\int \frac{\sinh^3(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=91

$$-\frac{3}{8}b\cosh(a)\text{Chi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\text{Chi}\left(3bx^2\right)-\frac{3}{8}b\sinh(a)\text{Shi}\left(bx^2\right)+\frac{3}{8}b\sinh(3a)\text{Shi}\left(3bx^2\right)+\frac{3\sinh\left(a+bx^2\right)}{8x^2}$$

```
[Out] -3/8*b*Chi(b*x^2)*cosh(a)+3/8*b*Chi(3*b*x^2)*cosh(3*a)-3/8*b*Shi(b*x^2)*sin
h(a)+3/8*b*Shi(3*b*x^2)*sinh(3*a)+3/8*sinh(b*x^2+a)/x^2-1/8*sinh(3*b*x^2+3*
a)/x^2
```

Rubi [A] time = 0.21, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5340, 5320, 3297, 3303, 3298, 3301}

$$-\frac{3}{8}b\cosh(a)\text{Chi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\text{Chi}\left(3bx^2\right)-\frac{3}{8}b\sinh(a)\text{Shi}\left(bx^2\right)+\frac{3}{8}b\sinh(3a)\text{Shi}\left(3bx^2\right)+\frac{3\sinh\left(a+bx^2\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^2])/8 + (3*b*Cosh[3*a]*CoshIntegral[3*b*x^2]
)/8 + (3*Sinh[a + b*x^2])/(8*x^2) - Sinh[3*(a + b*x^2)]/(8*x^2) - (3*b*Sinh
[a]*SinhIntegral[b*x^2])/8 + (3*b*Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx &= \int \left(-\frac{3\sinh\left(a+bx^2\right)}{4x^3}+\frac{\sinh\left(3a+3bx^2\right)}{4x^3}\right)dx\\
&= \frac{1}{4}\int \frac{\sinh\left(3a+3bx^2\right)}{x^3}\,dx-\frac{3}{4}\int \frac{\sinh\left(a+bx^2\right)}{x^3}\,dx\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{\sinh(3a+3bx)}{x^2}\,dx,x,x^2\right)-\frac{3}{8}\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,x^2\right)\\
&= \frac{3\sinh\left(a+bx^2\right)}{8x^2}-\frac{\sinh\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,x^2\right)+\frac{1}{8}(3b
\\
&= \frac{3\sinh\left(a+bx^2\right)}{8x^2}-\frac{\sinh\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{1}{8}(3b\cosh(a))\operatorname{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,x^2\right)+
\\
&= -\frac{3}{8}b\cosh(a)\operatorname{Chi}\left(bx^2\right)+\frac{3}{8}b\cosh(3a)\operatorname{Chi}\left(3bx^2\right)+\frac{3\sinh\left(a+bx^2\right)}{8x^2}-\frac{\sinh\left(3\left(a+bx^2\right)\right.}{8x^2}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 90, normalized size = 0.99

$$-\frac{3bx^2\cosh(a)\operatorname{Chi}\left(bx^2\right)-3bx^2\cosh(3a)\operatorname{Chi}\left(3bx^2\right)+3bx^2\sinh(a)\operatorname{Shi}\left(bx^2\right)-3bx^2\sinh(3a)\operatorname{Shi}\left(3bx^2\right)-3\sinh}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^2]^3/x^3,x]
```

```
[Out] -1/8*(3*b*x^2*Cosh[a]*CoshIntegral[b*x^2] - 3*b*x^2*Cosh[3*a]*CoshIntegral[
3*b*x^2] - 3*Sinh[a + b*x^2] + Sinh[3*(a + b*x^2)] + 3*b*x^2*Sinh[a]*SinhIn
tegral[b*x^2] - 3*b*x^2*Sinh[3*a]*SinhIntegral[3*b*x^2])/x^2
```

fricas [A] time = 0.50, size = 160, normalized size = 1.76

$$-\frac{2\sinh\left(bx^2+a\right)^3-3\left(bx^2\mathrm{Ei}\left(3\,bx^2\right)+bx^2\mathrm{Ei}\left(-3\,bx^2\right)\right)\cosh\left(3\,a\right)+3\left(bx^2\mathrm{Ei}\left(bx^2\right)+bx^2\mathrm{Ei}\left(-bx^2\right)\right)\cosh(a)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*sinh(b*x^2 + a)^3 - 3*(b*x^2*Ei(3*b*x^2) + b*x^2*Ei(-3*b*x^2))*cos
h(3*a) + 3*(b*x^2*Ei(b*x^2) + b*x^2*Ei(-b*x^2))*cosh(a) + 6*(cosh(b*x^2 + a
)^2 - 1)*sinh(b*x^2 + a) - 3*(b*x^2*Ei(3*b*x^2) - b*x^2*Ei(-3*b*x^2))*sinh(
3*a) + 3*(b*x^2*Ei(b*x^2) - b*x^2*Ei(-b*x^2))*sinh(a))/x^2
```

giac [B] time = 0.19, size = 223, normalized size = 2.45

$$\frac{3\left(bx^2+a\right)b^2\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}-3\,ab^2\mathrm{Ei}\left(3\,bx^2\right)e^{(3\,a)}-3\left(bx^2+a\right)b^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+3\,ab^2\mathrm{Ei}\left(-bx^2\right)e^{(-a)}+3\left(bx^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*(b*x^2 + a)*b^2*Ei(3*b*x^2)*e^(3*a) - 3*a*b^2*Ei(3*b*x^2)*e^(3*a) -
 3*(b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) + 3*a*b^2*Ei(-b*x^2)*e^(-a) + 3*(b*x^2
 + a)*b^2*Ei(-3*b*x^2)*e^(-3*a) - 3*a*b^2*Ei(-3*b*x^2)*e^(-3*a) - 3*(b*x^2
+ a)*b^2*Ei(b*x^2)*e^a + 3*a*b^2*Ei(b*x^2)*e^a - b^2*e^(3*b*x^2 + 3*a) + 3*
b^2*e^(b*x^2 + a) - 3*b^2*e^(-b*x^2 - a) + b^2*e^(-3*b*x^2 - 3*a))/(b^2*x^2
)
```

maple [A] time = 0.08, size = 120, normalized size = 1.32

$$\frac{\mathrm{e}^{-3a}\mathrm{e}^{-3b\,x^2}}{16x^2}-\frac{3\,\mathrm{e}^{-3a}b\,\mathrm{Ei}\left(1,3b\,x^2\right)}{16}-\frac{3\,\mathrm{e}^{-a}\mathrm{e}^{-b\,x^2}}{16x^2}+\frac{3\,\mathrm{e}^{-a}b\,\mathrm{Ei}\left(1,b\,x^2\right)}{16}-\frac{\mathrm{e}^{3a}\mathrm{e}^{3b\,x^2}}{16x^2}-\frac{3\,\mathrm{e}^{3a}b\,\mathrm{Ei}\left(1,-3b\,x^2\right)}{16}+\frac{3\,\mathrm{e}^{a}\mathrm{e}^{b\,x^2}}{16x^2}+\frac{3\,\mathrm{e}^{a}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(b*x^2+a)^3/x^3,x)
```

```
[Out] 1/16*exp(-3*a)/x^2*exp(-3*b*x^2)-3/16*exp(-3*a)*b*Ei(1,3*b*x^2)-3/16*exp(-a
)/x^2*exp(-b*x^2)+3/16*exp(-a)*b*Ei(1,b*x^2)-1/16*exp(3*a)/x^2*exp(3*b*x^2)
-3/16*exp(3*a)*b*Ei(1,-3*b*x^2)+3/16*exp(a)*exp(b*x^2)/x^2+3/16*exp(a)*b*Ei
(1,-b*x^2)
```

maxima [A] time = 0.43, size = 58, normalized size = 0.64

$$\frac{3}{16}\,be^{(-3\,a)}\Gamma\left(-1,3\,bx^2\right)-\frac{3}{16}\,be^{(-a)}\Gamma\left(-1,bx^2\right)-\frac{3}{16}\,be^{a}\Gamma\left(-1,-bx^2\right)+\frac{3}{16}\,be^{(3\,a)}\Gamma\left(-1,-3\,bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/16*b*e^(-3*a)*gamma(-1, 3*b*x^2) - 3/16*b*e^(-a)*gamma(-1, b*x^2) - 3/16*
b*e^a*gamma(-1, -b*x^2) + 3/16*b*e^(3*a)*gamma(-1, -3*b*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(b\,x^2+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^3/x^3,x)
```

```
[Out] int(sinh(a + b*x^2)^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**2)**3/x**3, x)
```

3.22 $\int x \sinh^7(a+bx^2)\,dx$

Optimal. Leaf size=67

$$\frac{\cosh^7(a+bx^2)}{14b}-\frac{3\cosh^5(a+bx^2)}{10b}+\frac{\cosh^3(a+bx^2)}{2b}-\frac{\cosh(a+bx^2)}{2b}$$

```
[Out] -1/2*cosh(b*x^2+a)/b+1/2*cosh(b*x^2+a)^3/b-3/10*cosh(b*x^2+a)^5/b+1/14*cosh
(b*x^2+a)^7/b
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2633}

$$\frac{\cosh^7(a+bx^2)}{14b}-\frac{3\cosh^5(a+bx^2)}{10b}+\frac{\cosh^3(a+bx^2)}{2b}-\frac{\cosh(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^2]^7,x]
```

```
[Out] -Cosh[a + b*x^2]/(2*b) + Cosh[a + b*x^2]^3/(2*b) - (3*Cosh[a + b*x^2]^5)/(1
0*b) + Cosh[a + b*x^2]^7/(14*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x\sinh^7\left(a+bx^2\right)\,dx=\frac{1}{2}\operatorname{Subst}\left(\int\sinh^7(a+bx)\,dx,x,x^2\right)$$
$$=-\frac{\operatorname{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,\cosh\left(a+bx^2\right)\right)}{2b}$$
$$=-\frac{\cosh\left(a+bx^2\right)}{2b}+\frac{\cosh^3\left(a+bx^2\right)}{2b}-\frac{3\cosh^5\left(a+bx^2\right)}{10b}+\frac{\cosh^7\left(a+bx^2\right)}{14b}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.00

$$-\frac{35\cosh\left(a+bx^2\right)}{128b}+\frac{7\cosh\left(3\left(a+bx^2\right)\right)}{128b}-\frac{7\cosh\left(5\left(a+bx^2\right)\right)}{640b}+\frac{\cosh\left(7\left(a+bx^2\right)\right)}{896b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^2]^7,x]
```

```
[Out] (-35*Cosh[a + b*x^2])/(128*b) + (7*Cosh[3*(a + b*x^2)])/(128*b) - (7*Cosh[5
*(a + b*x^2)])/(640*b) + Cosh[7*(a + b*x^2)]/(896*b)
```

fricas [B] time = 0.42, size = 154, normalized size = 2.30

$$\frac{5\cosh\left(bx^2+a\right)^7+35\cosh\left(bx^2+a\right)\sinh\left(bx^2+a\right)^6-49\cosh\left(bx^2+a\right)^5+35\left(5\cosh\left(bx^2+a\right)^3-7\cosh\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/4480*(5*cosh(b*x^2 + a)^7 + 35*cosh(b*x^2 + a)*sinh(b*x^2 + a)^6 - 49*cos
h(b*x^2 + a)^5 + 35*(5*cosh(b*x^2 + a)^3 - 7*cosh(b*x^2 + a))*sinh(b*x^2 +
a)^4 + 245*cosh(b*x^2 + a)^3 + 35*(3*cosh(b*x^2 + a)^5 - 14*cosh(b*x^2 + a)
^3 + 21*cosh(b*x^2 + a))*sinh(b*x^2 + a)^2 - 1225*cosh(b*x^2 + a))/b
```

giac [A] time = 0.23, size = 108, normalized size = 1.61

$$-\frac{\left(1225\,e^{\left(6\,bx^2+6\,a\right)}-245\,e^{\left(4\,bx^2+4\,a\right)}+49\,e^{\left(2\,bx^2+2\,a\right)}-5\right)e^{\left(-7\,bx^2-7\,a\right)}-5\,e^{\left(7\,bx^2+7\,a\right)}+49\,e^{\left(5\,bx^2+5\,a\right)}-245\,e^{\left(3\,bx^2+3\,a\right)}+}{8960\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/8960*((1225*e^(6*b*x^2 + 6*a) - 245*e^(4*b*x^2 + 4*a) + 49*e^(2*b*x^2 +
2*a) - 5)*e^(-7*b*x^2 - 7*a) - 5*e^(7*b*x^2 + 7*a) + 49*e^(5*b*x^2 + 5*a) -
 245*e^(3*b*x^2 + 3*a) + 1225*e^(b*x^2 + a))/b
```

maple [A] time = 0.12, size = 52, normalized size = 0.78

$$\frac{\left(-\frac{16}{35}+\frac{\left(\sinh^{6}\left(b\,x^{2}+a\right)\right)}{7}-\frac{6\left(\sinh^{4}\left(b\,x^{2}+a\right)\right)}{35}+\frac{8\left(\sinh^{2}\left(b\,x^{2}+a\right)\right)}{35}\right)\cosh\left(b\,x^{2}+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(b*x^2+a)^7,x)
```

```
[Out] 1/2/b*(-16/35+1/7*sinh(b*x^2+a)^6-6/35*sinh(b*x^2+a)^4+8/35*sinh(b*x^2+a)^2
)*cosh(b*x^2+a)
```

maxima [B] time = 0.42, size = 126, normalized size = 1.88

$$\frac{e^{\left(7\,bx^{2}+7\,a\right)}}{1792\,b}-\frac{7\,e^{\left(5\,bx^{2}+5\,a\right)}}{1280\,b}+\frac{7\,e^{\left(3\,bx^{2}+3\,a\right)}}{256\,b}-\frac{35\,e^{\left(bx^{2}+a\right)}}{256\,b}-\frac{35\,e^{\left(-bx^{2}-a\right)}}{256\,b}+\frac{7\,e^{\left(-3\,bx^{2}-3\,a\right)}}{256\,b}-\frac{7\,e^{\left(-5\,bx^{2}-5\,a\right)}}{1280\,b}+\frac{e^{\left(-7\,bx^{2}-7\,a\right)}}{1792\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/1792*e^(7*b*x^2 + 7*a)/b - 7/1280*e^(5*b*x^2 + 5*a)/b + 7/256*e^(3*b*x^2
+ 3*a)/b - 35/256*e^(b*x^2 + a)/b - 35/256*e^(-b*x^2 - a)/b + 7/256*e^(-3*b
*x^2 - 3*a)/b - 7/1280*e^(-5*b*x^2 - 5*a)/b + 1/1792*e^(-7*b*x^2 - 7*a)/b
```

mupad [B] time = 0.47, size = 52, normalized size = 0.78

$$-\frac{-5\cosh\left(b\,x^{2}+a\right)^{7}+21\cosh\left(b\,x^{2}+a\right)^{5}-35\cosh\left(b\,x^{2}+a\right)^{3}+35\cosh\left(b\,x^{2}+a\right)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^2)^7,x)
```

```
[Out] -(35*cosh(a + b*x^2) - 35*cosh(a + b*x^2)^3 + 21*cosh(a + b*x^2)^5 - 5*cosh
(a + b*x^2)^7)/(70*b)
```

sympy [A] time = 7.69, size = 94, normalized size = 1.40

$$\begin{cases} \frac{\sinh^{6}\left(a+bx^{2}\right)\cosh\left(a+bx^{2}\right)}{2b}-\frac{\sinh^{4}\left(a+bx^{2}\right)\cosh^{3}\left(a+bx^{2}\right)}{b}+\frac{4\sinh^{2}\left(a+bx^{2}\right)\cosh^{5}\left(a+bx^{2}\right)}{5b}-\frac{8\cosh^{7}\left(a+bx^{2}\right)}{35b} & \text{for}\: b\neq 0 \\ \frac{x^{2}\sinh^{7}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(b*x**2+a)**7,x)
```

```
[Out] Piecewise((sinh(a + b*x**2)**6*cosh(a + b*x**2)/(2*b) - sinh(a + b*x**2)**4
*cosh(a + b*x**2)**3/b + 4*sinh(a + b*x**2)**2*cosh(a + b*x**2)**5/(5*b) -
8*cosh(a + b*x**2)**7/(35*b), Ne(b, 0)), (x**2*sinh(a)**7/2, True))
```

3.23 $\int (ex)^m \sinh^p\left(a+bx^2\right) dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left((ex)^m \sinh^p\left(a+bx^2\right),x\right)$$

```
[Out] Unintegrable((e*x)^m*sinh(b*x^2+a)^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^p,x]
[Out] Defer[Int][(e*x)^m*Sinh[a + b*x^2]^p, x]
```

Rubi steps

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx=\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Mathematica [A] time = 2.51, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^p,x]
[Out] Integrate[(e*x)^m*Sinh[a + b*x^2]^p, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((ex)^m \sinh\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((e*x)^m*sinh(b*x^2 + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^p, x)
```

maple [A] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^p\left(b\,x^2+a\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a)^p,x)
[Out] int((e*x)^m*sinh(b*x^2+a)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sinh\left(b\,x^2+a\right)^p\,{\left(e\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^p*(e*x)^m,x)
[Out] int(sinh(a + b*x^2)^p*(e*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^p\left(a+bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**p,x)
[Out] Integral((e*x)**m*sinh(a + b*x**2)**p, x)
```

3.24 $\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=214

$$-\frac{e^{3a}3^{-\frac{m}{2}-\frac{1}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-3bx^2\right)}{16e}+\frac{3e^{a}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-bx^2\right)}{16e}-\frac{3e^{-a}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(e}{16}$$

```
[Out] -1/16*3^(-1/2-1/2*m)*exp(3*a)*(e*x)^(1+m)*(-b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1
/2*m,-3*b*x^2)/e+3/16*exp(a)*(e*x)^(1+m)*(-b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/
2*m,-b*x^2)/e-3/16*(e*x)^(1+m)*(b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,b*x^2)/
e/exp(a)+1/16*3^(-1/2-1/2*m)*(e*x)^(1+m)*(b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2
*m,3*b*x^2)/e/exp(3*a)
```

Rubi [A] time = 0.20, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5340, 5328, 2218}

$$-\frac{e^{3a}3^{-\frac{m}{2}-\frac{1}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-3bx^2\right)}{16e}+\frac{3e^{a}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)}{16e}-\frac{3e^{-a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^3,x]
```

```
[Out] -(3^(-1/2 - m/2)*E^(3*a)*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m
)/2, -3*b*x^2])/(16*e) + (3*E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma
[(1 + m)/2, -(b*x^2)])/(16*e) - (3*(e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma
[(1 + m)/2, b*x^2])/(16*e*E^a) + (3^(-1/2 - m/2)*(e*x)^(1 + m)*(b*x^2)^((-1
 - m)/2)*Gamma[(1 + m)/2, 3*b*x^2])/(16*e*E^(3*a))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^3\left(a+bx^2\right)\,dx &= \int \left(-\frac{3}{4}(ex)^m \sinh\left(a+bx^2\right)+\frac{1}{4}(ex)^m \sinh\left(3a+3bx^2\right)\right)dx\\ &= \frac{1}{4}\int (ex)^m \sinh\left(3a+3bx^2\right)dx-\frac{3}{4}\int (ex)^m \sinh\left(a+bx^2\right)dx\\ &= -\left(\frac{1}{8}\int e^{-3a-3bx^2}(ex)^m\,dx\right)+\frac{1}{8}\int e^{3a+3bx^2}(ex)^m\,dx+\frac{3}{8}\int e^{-a-bx^2}(ex)^m\,dx-\frac{3}{8}\int e^{a\cdot}\\ &= -\frac{3^{-\frac{1}{2}-\frac{m}{2}}e^{3a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-3bx^2\right)}{16e}+\frac{3e^a(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+\cdot}{2}\right.}{16e}\end{aligned}$$

Mathematica [B] time = 12.55, size = 735, normalized size = 3.43

$$\frac{1}{16}3^{\frac{1}{2}-\frac{m}{2}}x\sinh(a)\cosh^2(a)\left(-b^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left(\left(-bx^2\right)^{\frac{m+1}{2}}\left(3^{\frac{m+1}{2}}\Gamma\left(\frac{m+1}{2},bx^2\right)-\Gamma\left(\frac{m+1}{2},3bx^2\right)\right)-\left(bx^2\right)^{\frac{m+1}{2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^3,x]
```

```
[Out] ((e*x)^m*Cosh[a]^3*((-3*(-1/2*(x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 +
 m)/2, -(b*x^2)]) + (x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2]
)/2))/8 + (-1/2*(3^((-1 - m)/2)*x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1
+ m)/2, -3*b*x^2]) + (3^((-1 - m)/2)*x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(
1 + m)/2, 3*b*x^2])/2)/8))/x^m + (3^(1/2 - m/2)*x*(e*x)^m*(-(b^2*x^4))^((-1
 - m)/2)*Cosh[a]^2*(-((b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -3*b*x^2]) + 3^(
(1 + m)/2)*(b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -(b*x^2)] + (-(b*x^2))^((1
+ m)/2)*(3^((1 + m)/2)*Gamma[(1 + m)/2, b*x^2] - Gamma[(1 + m)/2, 3*b*x^2])
)*Sinh[a])/16 - (3^(1/2 - m/2)*x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*Cosh[a]*
((b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -3*b*x^2] + 3^((1 + m)/2)*(b*x^2)^((1
 + m)/2)*Gamma[(1 + m)/2, -(b*x^2)] - (-(b*x^2))^((1 + m)/2)*(3^((1 + m)/2)
*Gamma[(1 + m)/2, b*x^2] + Gamma[(1 + m)/2, 3*b*x^2]))*Sinh[a]^2)/16 + ((e*
x)^m*((3*(-1/2*(x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -(b*x^2)
]) - (x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/2))/8 + (-1/2
*(3^((-1 - m)/2)*x^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -3*b*x^
2]) - (3^((-1 - m)/2)*x^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, 3*b*x
^2])/2)/8)*Sinh[a]^3)/x^m
```

fricas [A] time = 0.59, size = 252, normalized size = 1.18

$$\frac{e\cosh\left(\frac{1}{2}\,(m-1)\log\left(\frac{3\,b}{e^2}\right)+3\,a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},3\,bx^2\right)-9\,e\cosh\left(\frac{1}{2}\,(m-1)\log\left(\frac{b}{e^2}\right)+a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},bx^2\right)-9\,e\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(e*cosh(1/2*(m - 1)*log(3*b/e^2) + 3*a)*gamma(1/2*m + 1/2, 3*b*x^2) -
9*e*cosh(1/2*(m - 1)*log(b/e^2) + a)*gamma(1/2*m + 1/2, b*x^2) - 9*e*cosh(1
/2*(m - 1)*log(-b/e^2) - a)*gamma(1/2*m + 1/2, -b*x^2) + e*cosh(1/2*(m - 1)
*log(-3*b/e^2) - 3*a)*gamma(1/2*m + 1/2, -3*b*x^2) - e*gamma(1/2*m + 1/2, 3
*b*x^2)*sinh(1/2*(m - 1)*log(3*b/e^2) + 3*a) + 9*e*gamma(1/2*m + 1/2, b*x^2
)*sinh(1/2*(m - 1)*log(b/e^2) + a) + 9*e*gamma(1/2*m + 1/2, -b*x^2)*sinh(1/
2*(m - 1)*log(-b/e^2) - a) - e*gamma(1/2*m + 1/2, -3*b*x^2)*sinh(1/2*(m - 1
)*log(-3*b/e^2) - 3*a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^3, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^3\left(b\,x^2+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a)^3,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(b\,x^2+a\right)^3\,{\left(e\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^3*(e*x)^m,x)
```

```
[Out] int(sinh(a + b*x^2)^3*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{m} \sinh^{3}{\left(a + bx^{2} \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**3, x)
```

3.25 $\int (ex)^m \sinh^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=135

$$-\frac{e^{2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-2bx^2\right)}{e}-\frac{e^{-2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},2bx^2\right)}{e}-\frac{(ex)^{m+1}}{2e(m+1)}$$

```
[Out] -1/2*(e*x)^(1+m)/e/(1+m)-2^(-7/2-1/2*m)*exp(2*a)*(e*x)^(1+m)*(-b*x^2)^(-1/2
-1/2*m)*GAMMA(1/2+1/2*m,-2*b*x^2)/e-2^(-7/2-1/2*m)*(e*x)^(1+m)*(b*x^2)^(-1/
2-1/2*m)*GAMMA(1/2+1/2*m,2*b*x^2)/e/exp(2*a)
```

Rubi [A] time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5340, 5329, 2218}

$$-\frac{e^{2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-2bx^2\right)}{e}-\frac{e^{-2a}2^{-\frac{m}{2}-\frac{7}{2}}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},2bx^2\right)}{e}-$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2]^2,x]
```

```
[Out] -(e*x)^(1 + m)/(2*e*(1 + m)) - (2^(-7/2 - m/2)*E^(2*a)*(e*x)^(1 + m)*(-(b*x
^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -2*b*x^2])/e - (2^(-7/2 - m/2)*(e*x)^(1
+ m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, 2*b*x^2])/(e*E^(2*a))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5329

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \sinh^2\left(a+bx^2\right)\,dx = \int \left(-\frac{1}{2}(ex)^m + \frac{1}{2}(ex)^m \cosh\left(2a+2bx^2\right)\right)dx$$
$$= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{2}\int (ex)^m \cosh\left(2a+2bx^2\right)dx$$
$$= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{4}\int e^{-2a-2bx^2}(ex)^m\,dx + \frac{1}{4}\int e^{2a+2bx^2}(ex)^m\,dx$$
$$= -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{2a}(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-2bx^2\right)}{e} - \frac{2^{-\frac{7}{2}-\frac{m}{2}}e^{-2a}(ex)^{1+m}}{}$$

Mathematica [A] time = 0.58, size = 152, normalized size = 1.13

$$-\frac{2^{\frac{1}{2}(-m-7)}x\left(-b^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left((m+1)(\cosh(2a)-\sinh(2a))\left(-bx^2\right)^{\frac{m+1}{2}}\Gamma\left(\frac{m+1}{2},2bx^2\right)+(m+1)(\sinh(2a)+\mathrm{c}\right.}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2]^2,x]
```

```
[Out] -((2^((-7 - m)/2)*x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*(2^((5 + m)/2)*(-(b^2
*x^4))^((1 + m)/2) + (1 + m)*(-(b*x^2))^((1 + m)/2)*Gamma[(1 + m)/2, 2*b*x^
2]*(Cosh[2*a] - Sinh[2*a]) + (1 + m)*(b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2, -
2*b*x^2]*(Cosh[2*a] + Sinh[2*a])))/(1 + m))
```

fricas [A] time = 0.75, size = 174, normalized size = 1.29

$$-\frac{8\,bx\cosh\left(m\log\left(ex\right)\right)+\left(em+e\right)\cosh\left(\frac{1}{2}\left(m-1\right)\log\left(\frac{2\,b}{e^2}\right)+2\,a\right)\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},2\,bx^2\right)-\left(em+e\right)\cosh\left(\frac{1}{2}\left(m-1\right)\mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*b*x*cosh(m*log(e*x)) + (e*m + e)*cosh(1/2*(m - 1)*log(2*b/e^2) + 2
*a)*gamma(1/2*m + 1/2, 2*b*x^2) - (e*m + e)*cosh(1/2*(m - 1)*log(-2*b/e^2)
- 2*a)*gamma(1/2*m + 1/2, -2*b*x^2) + 8*b*x*sinh(m*log(e*x)) - (e*m + e)*ga
```

```
mma(1/2*m + 1/2, 2*b*x^2)*sinh(1/2*(m - 1)*log(2*b/e^2) + 2*a) + (e*m + e)*
gamma(1/2*m + 1/2, -2*b*x^2)*sinh(1/2*(m - 1)*log(-2*b/e^2) - 2*a))/(b*m +
b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2 + a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^2\left(b\,x^2 + a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a)^2,x)
```

```
[Out] int((e*x)^m*sinh(b*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,e^m \int e^{\left(2\,bx^2 + m\log(x) + 2\,a\right)}\,dx + \frac{1}{4}\,e^m \int e^{\left(-2\,bx^2 + m\log(x) - 2\,a\right)}\,dx - \frac{(ex)^{m+1}}{2\,e(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*e^m*integrate(e^(2*b*x^2 + m*log(x) + 2*a), x) + 1/4*e^m*integrate(e^(-
2*b*x^2 + m*log(x) - 2*a), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(b\,x^2 + a\right)^2 (e\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)^2*(e*x)^m,x)
```

```
[Out] int(sinh(a + b*x^2)^2*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^2\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2)**2, x)
```

3.26 $\int (ex)^m \sinh\left(a+bx^2\right)\,dx$

Optimal. Leaf size=95

$$\frac{e^{-a}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},bx^2\right)}{4e}-\frac{e^{a}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\Gamma\left(\frac{m+1}{2},-bx^2\right)}{4e}$$

```
[Out] -1/4*exp(a)*(e*x)^(1+m)*(-b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-b*x^2)/e+1/4
*(e*x)^(1+m)*(b*x^2)^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,b*x^2)/e/exp(a)
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5328, 2218}

$$\frac{e^{-a}\left(bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},bx^2\right)}{4e}-\frac{e^{a}\left(-bx^2\right)^{\frac{1}{2}(-m-1)}(ex)^{m+1}\text{Gamma}\left(\frac{m+1}{2},-bx^2\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^2],x]
```

```
[Out] -(E^a*(e*x)^(1 + m)*(-(b*x^2))^((-1 - m)/2)*Gamma[(1 + m)/2, -(b*x^2)])/(4*
e) + ((e*x)^(1 + m)*(b*x^2)^((-1 - m)/2)*Gamma[(1 + m)/2, b*x^2])/(4*e*E^a)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int (ex)^m \sinh\left(a+bx^2\right)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^2}(ex)^m\,dx\right) + \frac{1}{2}\int e^{a+bx^2}(ex)^m\,dx$$
$$= -\frac{e^a(ex)^{1+m}\left(-bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},-bx^2\right)}{4e} + \frac{e^{-a}(ex)^{1+m}\left(bx^2\right)^{\frac{1}{2}(-1-m)}\Gamma\left(\frac{1+m}{2},bx^2\right)}{4e}$$

Mathematica [A] time = 0.15, size = 98, normalized size = 1.03

$$-\frac{1}{4}x\left(-b^2x^4\right)^{\frac{1}{2}(-m-1)}(ex)^m\left((\sinh(a)+\cosh(a))\left(bx^2\right)^{\frac{m+1}{2}}\Gamma\left(\frac{m+1}{2},-bx^2\right)-(\cosh(a)-\sinh(a))\left(-bx^2\right)^{\frac{m+1}{2}}\Gamma\left(\frac{m+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^2],x]
```

```
[Out] -1/4*(x*(e*x)^m*(-(b^2*x^4))^((-1 - m)/2)*(-((-(b*x^2))^((1 + m)/2)*Gamma[(
1 + m)/2, b*x^2]*(Cosh[a] - Sinh[a])) + (b*x^2)^((1 + m)/2)*Gamma[(1 + m)/2
, -(b*x^2)]*(Cosh[a] + Sinh[a])))
```

fricas [A] time = 0.45, size = 124, normalized size = 1.31

$$\frac{e\cosh\left(\frac{1}{2}(m-1)\log\left(\frac{b}{e^2}\right)+a\right)\Gamma\left(\frac{1}{2}m+\frac{1}{2},bx^2\right)+e\cosh\left(\frac{1}{2}(m-1)\log\left(-\frac{b}{e^2}\right)-a\right)\Gamma\left(\frac{1}{2}m+\frac{1}{2},-bx^2\right)-e\Gamma\left(\frac{1}{2}m+\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(e*cosh(1/2*(m - 1)*log(b/e^2) + a)*gamma(1/2*m + 1/2, b*x^2) + e*cosh(
1/2*(m - 1)*log(-b/e^2) - a)*gamma(1/2*m + 1/2, -b*x^2) - e*gamma(1/2*m + 1
/2, b*x^2)*sinh(1/2*(m - 1)*log(b/e^2) + a) - e*gamma(1/2*m + 1/2, -b*x^2)*
sinh(1/2*(m - 1)*log(-b/e^2) - a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a), x)
```

maple [C] time = 0.10, size = 77, normalized size = 0.81

$$\frac{(ex)^m x \,\text{hypergeom}\left(\left[\frac{m}{4}+\frac{1}{4}\right],\left[\frac{1}{2},\frac{5}{4}+\frac{m}{4}\right],\frac{x^4b^2}{4}\right)\sinh(a)}{1+m}+\frac{(ex)^m b\, x^3 \,\text{hypergeom}\left(\left[\frac{3}{4}+\frac{m}{4}\right],\left[\frac{3}{2},\frac{7}{4}+\frac{m}{4}\right],\frac{x^4b^2}{4}\right)\text{co}}{3+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(b*x^2+a),x)
```

```
[Out] (e*x)^m/(1+m)*x*hypergeom([1/4*m+1/4],[1/2,5/4+1/4*m],1/4*x^4*b^2)*sinh(a)+
(e*x)^m*b/(3+m)*x^3*hypergeom([3/4+1/4*m],[3/2,7/4+1/4*m],1/4*x^4*b^2)*cosh
(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(b\,x^2+a\right)\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^2)*(e*x)^m,x)
```

```
[Out] int(sinh(a + b*x^2)*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(b*x**2+a),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**2), x)
```

3.27 $\int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=26

$$x^{-m}(ex)^m \operatorname{Int}\left(x^m \operatorname{csch}\left(a+bx^2\right),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(b*x^2+a),x)/(x^m)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b*x^2],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b*x^2], x])/x^m
```

Rubi steps

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx = \left(x^{-m}(ex)^m\right)\int x^m \operatorname{csch}\left(a+bx^2\right)\,dx$$

Mathematica [A] time = 2.96, size = 0, normalized size = 0.00

$$\int (ex)^m \operatorname{csch}\left(a+bx^2\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b*x^2],x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b*x^2], x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(ex)^m}{\sinh\left(bx^2+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/sinh(b*x^2 + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(b*x^2 + a), x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(b*x^2+a),x)
```

```
[Out] int((e*x)^m/sinh(b*x^2+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(bx^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/sinh(b*x^2 + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{\sinh\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a + b*x^2),x)
```

```
[Out] int((e*x)^m/sinh(a + b*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(b*x**2+a),x)
```

```
[Out] Integral((e*x)**m/sinh(a + b*x**2), x)
```

3.28 $\int x^3 \sinh\left(a+bx^4\right)\,dx$

Optimal. Leaf size=15

$$\frac{\cosh\left(a+bx^4\right)}{4b}$$

```
[Out] 1/4*cosh(b*x^4+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$\frac{\cosh\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b*x^4],x]
```

```
[Out] Cosh[a + b*x^4]/(4*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3 \sinh\left(a+bx^4\right)\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \sinh(a+bx)\,dx, x, x^4\right)\\ &= \frac{\cosh\left(a+bx^4\right)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cosh\left(a+bx^4\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b*x^4],x]
[Out] Cosh[a + b*x^4]/(4*b)
```

fricas [A] time = 0.42, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx^4 + a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="fricas")
[Out] 1/4*cosh(b*x^4 + a)/b
```

giac [A] time = 0.22, size = 25, normalized size = 1.67

$$\frac{e^{\left(bx^4+a\right)} + e^{\left(-bx^4-a\right)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="giac")
[Out] 1/8*(e^(b*x^4 + a) + e^(-b*x^4 - a))/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\cosh\left(b\,x^4 + a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(b*x^4+a),x)
[Out] 1/4*cosh(b*x^4+a)/b
```

maxima [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{\cosh\left(bx^4 + a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*cosh(b*x^4 + a)/b
```

mupad [B] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\cosh\left(b\,x^4+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a + b*x^4),x)
[Out] cosh(a + b*x^4)/(4*b)
```

sympy [A] time = 0.77, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\cosh\left(a+bx^4\right)}{4b} & \text{for } b \neq 0 \\ \frac{x^4 \sinh\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(b*x**4+a),x)
[Out] Piecewise((cosh(a + b*x**4)/(4*b), Ne(b, 0)), (x**4*sinh(a)/4, True))
```

3.29 $\int x^2 \sinh\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=78

$$-\frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)$$

```
[Out] -1/6*b^3*Chi(b/x)*cosh(a)+1/6*b*x^2*cosh(a+b/x)-1/6*b^3*Shi(b/x)*sinh(a)+1/
6*b^2*x*sinh(a+b/x)+1/3*x^3*sinh(a+b/x)
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-\frac{1}{6}b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+\frac{1}{6}b^2x\sinh\left(a+\frac{b}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}bx^2\cosh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b/x],x]
```

```
[Out] (b*x^2*Cosh[a + b/x])/6 - (b^3*Cosh[a]*CoshIntegral[b/x])/6 + (b^2*x*Sinh[a
 + b/x])/6 + (x^3*Sinh[a + b/x])/3 - (b^3*Sinh[a]*SinhIntegral[b/x])/6
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^2 \sinh\left(a+\frac{b}{x}\right) dx &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^4} dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x}\right) - \frac{1}{3}b\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^3} dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}bx^2 \cosh\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x}\right) - \frac{1}{6}b^2\,\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2} dx, x, \frac{1}{x}\right) \\
&= \frac{1}{6}bx^2 \cosh\left(a+\frac{b}{x}\right) + \frac{1}{6}b^2 x \sinh\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x}\right) - \frac{1}{6}b^3\,\text{Subst}\left(\int \frac{\cosh(a-}{x}\right. \\
&= \frac{1}{6}bx^2 \cosh\left(a+\frac{b}{x}\right) + \frac{1}{6}b^2 x \sinh\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x}\right) - \frac{1}{6}\left(b^3\cosh(a)\right)\text{Subst}\left(\right. \\
&= \frac{1}{6}bx^2 \cosh\left(a+\frac{b}{x}\right) - \frac{1}{6}b^3\cosh(a)\text{Chi}\left(\frac{b}{x}\right) + \frac{1}{6}b^2 x \sinh\left(a+\frac{b}{x}\right) + \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x}\right) -
\end{aligned}$$

Mathematica [A] time = 0.07, size = 70, normalized size = 0.90

$$\frac{1}{6}\left(b^3(-\cosh(a))\text{Chi}\left(\frac{b}{x}\right) - b^3\sinh(a)\text{Shi}\left(\frac{b}{x}\right) + x\left(b^2\sinh\left(a+\frac{b}{x}\right) + 2x^2\sinh\left(a+\frac{b}{x}\right) + bx\cosh\left(a+\frac{b}{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b/x],x]
```

```
[Out] (-(b^3*Cosh[a]*CoshIntegral[b/x]) + x*(b*x*Cosh[a + b/x] + b^2*Sinh[a + b/x
] + 2*x^2*Sinh[a + b/x]) - b^3*Sinh[a]*SinhIntegral[b/x])/6
```

fricas [A] time = 0.42, size = 93, normalized size = 1.19

$$\frac{1}{6}\,bx^2\cosh\left(\frac{ax+b}{x}\right)-\frac{1}{12}\left(b^3\mathrm{Ei}\left(\frac{b}{x}\right)+b^3\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a)-\frac{1}{12}\left(b^3\mathrm{Ei}\left(\frac{b}{x}\right)-b^3\mathrm{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)+\frac{1}{6}\left(b^2x+2\,x^3\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^2*cosh((a*x + b)/x) - 1/12*(b^3*Ei(b/x) + b^3*Ei(-b/x))*cosh(a) - 1
/12*(b^3*Ei(b/x) - b^3*Ei(-b/x))*sinh(a) + 1/6*(b^2*x + 2*x^3)*sinh((a*x +
b)/x)
```

giac [B] time = 0.39, size = 534, normalized size = 6.85

$$-\frac{a^3b^4\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}+a^3b^4\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}-\frac{3\,(ax+b)a^2b^4\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}}{x}-\frac{3\,(ax+b)a^2b^4\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{x}+\frac{3\,(ax+b)^2ab^4\mathrm{Ei}\left(a\right.}{x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="giac")
```

```
[Out] -1/12*(a^3*b^4*Ei(a - (a*x + b)/x)*e^(-a) + a^3*b^4*Ei(-a + (a*x + b)/x)*e^
a - 3*(a*x + b)*a^2*b^4*Ei(a - (a*x + b)/x)*e^(-a)/x - 3*(a*x + b)*a^2*b^4*
Ei(-a + (a*x + b)/x)*e^a/x + 3*(a*x + b)^2*a*b^4*Ei(a - (a*x + b)/x)*e^(-a)
/x^2 + 3*(a*x + b)^2*a*b^4*Ei(-a + (a*x + b)/x)*e^a/x^2 + a^2*b^4*e^((a*x +
 b)/x) - a^2*b^4*e^(-(a*x + b)/x) - (a*x + b)^3*b^4*Ei(a - (a*x + b)/x)*e^(
-a)/x^3 - (a*x + b)^3*b^4*Ei(-a + (a*x + b)/x)*e^a/x^3 - a*b^4*e^((a*x + b)
/x) - 2*(a*x + b)*a*b^4*e^((a*x + b)/x)/x - a*b^4*e^(-(a*x + b)/x) + 2*(a*x
 + b)*a*b^4*e^(-(a*x + b)/x)/x + 2*b^4*e^((a*x + b)/x) + (a*x + b)^2*b^4*e^
((a*x + b)/x)/x^2 + (a*x + b)*b^4*e^((a*x + b)/x)/x - 2*b^4*e^(-(a*x + b)/x
) - (a*x + b)^2*b^4*e^(-(a*x + b)/x)/x^2 + (a*x + b)*b^4*e^(-(a*x + b)/x)/x
)/((a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3)*b)
```

maple [A] time = 0.06, size = 130, normalized size = 1.67

$$-\frac{b^2\mathrm{e}^{-\frac{ax+b}{x}}x}{12}+\frac{b\,\mathrm{e}^{-\frac{ax+b}{x}}x^2}{12}-\frac{\mathrm{e}^{-\frac{ax+b}{x}}x^3}{6}+\frac{b^3\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,\frac{b}{x}\right)}{12}+\frac{\mathrm{e}^{\frac{ax+b}{x}}x^3}{6}+\frac{b\,\mathrm{e}^{\frac{ax+b}{x}}x^2}{12}+\frac{b^2\mathrm{e}^{\frac{ax+b}{x}}x}{12}+\frac{b^3\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-\frac{b}{x}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b/x),x)
```

```
[Out] -1/12*b^2*exp(-(a*x+b)/x)*x+1/12*b*exp(-(a*x+b)/x)*x^2-1/6*exp(-(a*x+b)/x)*
x^3+1/12*b^3*exp(-a)*Ei(1,b/x)+1/6*exp((a*x+b)/x)*x^3+1/12*b*exp((a*x+b)/x)
*x^2+1/12*b^2*exp((a*x+b)/x)*x+1/12*b^3*exp(a)*Ei(1,-b/x)
```

maxima [A] time = 0.37, size = 47, normalized size = 0.60

$$\frac{1}{3}\,x^3\sinh\left(a+\frac{b}{x}\right)+\frac{1}{6}\left(b^2e^{(-a)}\Gamma\left(-2,\frac{b}{x}\right)+b^2e^a\Gamma\left(-2,-\frac{b}{x}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(a + b/x) + 1/6*(b^2*e^(-a)*gamma(-2, b/x) + b^2*e^a*gamma(-2,
-b/x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sinh\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b/x),x)
```

```
[Out] int(x^2*sinh(a + b/x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b/x),x)
```

```
[Out] Integral(x**2*sinh(a + b/x), x)
```

3.30 $\int x \sinh\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=60

$$-\frac{1}{2}b^2\sinh(a)\mathrm{Chi}\left(\frac{b}{x}\right)-\frac{1}{2}b^2\cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)$$

```
[Out] 1/2*b*x*cosh(a+b/x)-1/2*b^2*cosh(a)*Shi(b/x)-1/2*b^2*Chi(b/x)*sinh(a)+1/2*x
^2*sinh(a+b/x)
```

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b^2\sinh(a)\mathrm{Chi}\left(\frac{b}{x}\right)-\frac{1}{2}b^2\cosh(a)\mathrm{Shi}\left(\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)+\frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b/x],x]
```

```
[Out] (b*x*Cosh[a + b/x])/2 - (b^2*CoshIntegral[b/x]*Sinh[a])/2 + (x^2*Sinh[a + b
/x])/2 - (b^2*Cosh[a]*SinhIntegral[b/x])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int x \sinh\left(a+\frac{b}{x}\right)dx &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}b\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}b^2\,\text{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}\left(b^2\cosh(a)\right)\text{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x}\right)-\\
&= \frac{1}{2}bx\cosh\left(a+\frac{b}{x}\right)-\frac{1}{2}b^2\text{Chi}\left(\frac{b}{x}\right)\sinh(a)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x}\right)-\frac{1}{2}b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{1}{2}\left(b^2\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right)-b^2\cosh(a)\text{Shi}\left(\frac{b}{x}\right)+x\left(x\sinh\left(a+\frac{b}{x}\right)+b\cosh\left(a+\frac{b}{x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b/x],x]
```

```
[Out] (-(b^2*CoshIntegral[b/x]*Sinh[a]) + x*(b*Cosh[a + b/x] + x*Sinh[a + b/x]) -
 b^2*Cosh[a]*SinhIntegral[b/x])/2
```

fricas [A] time = 0.42, size = 83, normalized size = 1.38

$$\frac{1}{2}bx\cosh\left(\frac{ax+b}{x}\right)+\frac{1}{2}x^2\sinh\left(\frac{ax+b}{x}\right)-\frac{1}{4}\left(b^2\text{Ei}\left(\frac{b}{x}\right)-b^2\text{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a)-\frac{1}{4}\left(b^2\text{Ei}\left(\frac{b}{x}\right)+b^2\text{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(a+b/x),x, algorithm="fricas")

[Out] 1/2*b*x*cosh((a*x + b)/x) + 1/2*x^2*sinh((a*x + b)/x) - 1/4*(b^2*Ei(b/x) - b^2*Ei(-b/x))*cosh(a) - 1/4*(b^2*Ei(b/x) + b^2*Ei(-b/x))*sinh(a)

giac [B] time = 0.22, size = 313, normalized size = 5.22

$$\frac{a^2b^3\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}-a^2b^3\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}-\frac{2\,(ax+b)ab^3\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}}{x}+\frac{2\,(ax+b)ab^3\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{x}+\frac{(ax+b)^2b^3\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)}{x^2}}{4\left(a^2-\frac{2\,(ax+b)a}{x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(a+b/x),x, algorithm="giac")

[Out] 1/4*(a^2*b^3*Ei(a - (a*x + b)/x)*e^(-a) - a^2*b^3*Ei(-a + (a*x + b)/x)*e^a - 2*(a*x + b)*a*b^3*Ei(a - (a*x + b)/x)*e^(-a)/x + 2*(a*x + b)*a*b^3*Ei(-a + (a*x + b)/x)*e^a/x + (a*x + b)^2*b^3*Ei(a - (a*x + b)/x)*e^(-a)/x^2 - (a*x + b)^2*b^3*Ei(-a + (a*x + b)/x)*e^a/x^2 - a*b^3*e^((a*x + b)/x) - a*b^3*e^(-(a*x + b)/x) + b^3*e^((a*x + b)/x) + (a*x + b)*b^3*e^((a*x + b)/x)/x - b^3*e^(-(a*x + b)/x) + (a*x + b)*b^3*e^(-(a*x + b)/x)/x)/((a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2)*b)

maple [A] time = 0.05, size = 93, normalized size = 1.55

$$\frac{b\,\mathrm{e}^{-\frac{ax+b}{x}}x}{4}-\frac{\mathrm{e}^{-\frac{ax+b}{x}}x^2}{4}-\frac{b^2\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,\frac{b}{x}\right)}{4}+\frac{\mathrm{e}^{\frac{ax+b}{x}}x^2}{4}+\frac{b\,\mathrm{e}^{\frac{ax+b}{x}}x}{4}+\frac{b^2\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-\frac{b}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sinh(a+b/x),x)

[Out] 1/4*b*exp(-(a*x+b)/x)*x-1/4*exp(-(a*x+b)/x)*x^2-1/4*b^2*exp(-a)*Ei(1,b/x)+1/4*exp((a*x+b)/x)*x^2+1/4*b*exp((a*x+b)/x)*x+1/4*b^2*exp(a)*Ei(1,-b/x)

maxima [A] time = 0.35, size = 44, normalized size = 0.73

$$\frac{1}{2}\,x^2\sinh\left(a+\frac{b}{x}\right)+\frac{1}{4}\left(be^{(-a)}\Gamma\left(-1,\frac{b}{x}\right)-be^{a}\Gamma\left(-1,-\frac{b}{x}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sinh(a+b/x),x, algorithm="maxima")

```
[Out] 1/2*x^2*sinh(a + b/x) + 1/4*(b*e^(-a)*gamma(-1, b/x) - b*e^a*gamma(-1, -b/x
))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \sinh\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b/x),x)
```

```
[Out] int(x*sinh(a + b/x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x),x)
```

```
[Out] Integral(x*sinh(a + b/x), x)
```

3.31 $\int \sinh\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=33

$$-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)$$

```
[Out] -b*Chi(b/x)*cosh(a)-b*Shi(b/x)*sinh(a)+x*sinh(a+b/x)
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5302, 3297, 3303, 3298, 3301}

$$-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)+x\sinh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sinh\left(a+\frac{b}{x}\right)dx &= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= x\sinh\left(a+\frac{b}{x}\right) - b\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\ &= x\sinh\left(a+\frac{b}{x}\right) - (b\cosh(a))\,\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, \frac{1}{x}\right) - (b\sinh(a))\,\text{Subst}\left(\int \frac{\sinh(b}{x}\right.\\ &= -b\cosh(a)\text{Chi}\left(\frac{b}{x}\right) + x\sinh\left(a+\frac{b}{x}\right) - b\sinh(a)\text{Shi}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-b\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - b\sinh(a)\text{Shi}\left(\frac{b}{x}\right) + x\sinh\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x]) + x*Sinh[a + b/x] - b*Sinh[a]*SinhIntegral[b
/x]
```

fricas [A] time = 0.44, size = 58, normalized size = 1.76

$$-\frac{1}{2}\left(b\text{Ei}\left(\frac{b}{x}\right) + b\text{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a) - \frac{1}{2}\left(b\text{Ei}\left(\frac{b}{x}\right) - b\text{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a) + x\sinh\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x, algorithm="fricas")
```

```
[Out] -1/2*(b*Ei(b/x) + b*Ei(-b/x))*cosh(a) - 1/2*(b*Ei(b/x) - b*Ei(-b/x))*sinh(a
) + x*sinh((a*x + b)/x)
```

giac [B] time = 0.14, size = 173, normalized size = 5.24

$$-\frac{ab^2\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}-\frac{(ax+b)b^2\mathrm{Ei}\left(a-\frac{ax+b}{x}\right)e^{(-a)}}{x}-b^2e^{\left(-\frac{ax+b}{x}\right)}}{2\left(a-\frac{ax+b}{x}\right)b}-\frac{ab^2\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}-\frac{(ax+b)b^2\mathrm{Ei}\left(-a+\frac{ax+b}{x}\right)e^{a}}{x}+b^2e^{\left(\frac{ax+b}{x}\right)}}{2\left(a-\frac{ax+b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^2*Ei(a - (a*x + b)/x)*e^(-a) - (a*x + b)*b^2*Ei(a - (a*x + b)/x)*
e^(-a)/x - b^2*e^(-(a*x + b)/x))/((a - (a*x + b)/x)*b) - 1/2*(a*b^2*Ei(-a +
 (a*x + b)/x)*e^a - (a*x + b)*b^2*Ei(-a + (a*x + b)/x)*e^a/x + b^2*e^((a*x
+ b)/x))/((a - (a*x + b)/x)*b)
```

maple [A] time = 0.05, size = 56, normalized size = 1.70

$$\frac{b\,\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,\frac{b}{x}\right)}{2}-\frac{\mathrm{e}^{-\frac{ax+b}{x}}x}{2}+\frac{b\,\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-\frac{b}{x}\right)}{2}+\frac{\mathrm{e}^{\frac{ax+b}{x}}x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x),x)
```

```
[Out] 1/2*b*exp(-a)*Ei(1,b/x)-1/2*exp(-(a*x+b)/x)*x+1/2*b*exp(a)*Ei(1,-b/x)+1/2*e
xp((a*x+b)/x)*x
```

maxima [A] time = 0.36, size = 36, normalized size = 1.09

$$-\frac{1}{2}\left(\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)}+\mathrm{Ei}\left(\frac{b}{x}\right)e^{a}\right)b+x\sinh\left(a+\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x, algorithm="maxima")
```

```
[Out] -1/2*(Ei(-b/x)*e^(-a) + Ei(b/x)*e^a)*b + x*sinh(a + b/x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sinh\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x),x)
```

```
[Out] int(sinh(a + b/x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x),x)
```

```
[Out] Integral(sinh(a + b/x), x)
```

3.32 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x} \, dx$

Optimal. Leaf size=21

$$\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right)-\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[Out] -cosh(a)*Shi(b/x)-Chi(b/x)*sinh(a)
```

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5318, 5317, 5316}

$$\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right)-\cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x,x]
```

```
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x}\,dx = \cosh(a)\int \frac{\sinh\left(\frac{b}{x}\right)}{x}\,dx + \sinh(a)\int \frac{\cosh\left(\frac{b}{x}\right)}{x}\,dx$$
$$= -\text{Chi}\left(\frac{b}{x}\right)\sinh(a) - \cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\sinh(a)\left(-\text{Chi}\left(\frac{b}{x}\right)\right) - \cosh(a)\text{Shi}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x,x]
[Out] -(CoshIntegral[b/x]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x]
```

fricas [A] time = 0.40, size = 39, normalized size = 1.86

$$-\frac{1}{2}\left(\text{Ei}\left(\frac{b}{x}\right) - \text{Ei}\left(-\frac{b}{x}\right)\right)\cosh(a) - \frac{1}{2}\left(\text{Ei}\left(\frac{b}{x}\right) + \text{Ei}\left(-\frac{b}{x}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="fricas")
[Out] -1/2*(Ei(b/x) - Ei(-b/x))*cosh(a) - 1/2*(Ei(b/x) + Ei(-b/x))*sinh(a)
```

giac [B] time = 0.41, size = 44, normalized size = 2.10

$$\frac{b\text{Ei}\left(a - \frac{ax+b}{x}\right)e^{(-a)} - b\text{Ei}\left(-a + \frac{ax+b}{x}\right)e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="giac")
[Out] 1/2*(b*Ei(a - (a*x + b)/x)*e^(-a) - b*Ei(-a + (a*x + b)/x)*e^a)/b
```

maple [A] time = 0.04, size = 27, normalized size = 1.29

$$-\frac{e^{-a}\,\text{Ei}\left(1, \frac{b}{x}\right)}{2} + \frac{e^{a}\,\text{Ei}\left(1, -\frac{b}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x,x)
```

```
[Out] -1/2*exp(-a)*Ei(1,b/x)+1/2*exp(a)*Ei(1,-b/x)
```

maxima [A] time = 0.39, size = 24, normalized size = 1.14

$$\frac{1}{2}\,\mathrm{Ei}\left(-\frac{b}{x}\right)e^{(-a)}-\frac{1}{2}\,\mathrm{Ei}\left(\frac{b}{x}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x, algorithm="maxima")
```

```
[Out] 1/2*Ei(-b/x)*e^(-a) - 1/2*Ei(b/x)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$-\mathrm{sinh}(a)\,\mathrm{coshint}\left(\frac{b}{x}\right)-\mathrm{cosh}(a)\,\mathrm{sinhint}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)/x,x)
```

```
[Out] - sinh(a)*coshint(b/x) - cosh(a)*sinhint(b/x)
```

sympy [A] time = 1.03, size = 17, normalized size = 0.81

$$-\sinh{\left(a \right)}\operatorname{Chi}\left(\frac{b}{x}\right)-\cosh{\left(a \right)}\operatorname{Shi}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x,x)
```

```
[Out] -sinh(a)*Chi(b/x) - cosh(a)*Shi(b/x)
```

3.33 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2} dx$

Optimal. Leaf size=13

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -cosh(a+b/x)/b
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^2,x]
```

```
[Out] -(Cosh[a + b/x]/b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right) \\ &= -\frac{\cosh\left(a+\frac{b}{x}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^2,x]
[Out] -(Cosh[a + b/x]/b)
```

fricas [A] time = 0.42, size = 15, normalized size = 1.15

$$-\frac{\cosh\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="fricas")
[Out] -cosh((a*x + b)/x)/b
```

giac [B] time = 0.42, size = 27, normalized size = 2.08

$$-\frac{e^{\left(\frac{ax+b}{x}\right)}+e^{\left(-\frac{ax+b}{x}\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="giac")
[Out] -1/2*(e^((a*x + b)/x) + e^(-(a*x + b)/x))/b
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^2,x)
[Out] -cosh(a+b/x)/b
```

maxima [A] time = 0.38, size = 13, normalized size = 1.00

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] -cosh(a + b/x)/b
```

mupad [B] time = 0.37, size = 13, normalized size = 1.00

$$-\frac{\cosh\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)/x^2,x)
```

```
[Out] -cosh(a + b/x)/b
```

sympy [A] time = 0.98, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\sinh(a)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**2,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/b, Ne(b, 0)), (-sinh(a)/x, True))
```

3.34 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3} \, dx$

Optimal. Leaf size=29

$$\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}$$

```
[Out] -cosh(a+b/x)/b/x+sinh(a+b/x)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 3296, 2637}

$$\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^3,x]
```

```
[Out] -(Cosh[a + b/x]/(b*x)) + Sinh[a + b/x]/b^2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\text{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx} + \frac{\text{Subst}\left(\int\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx} + \frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{x\sinh\left(a+\frac{b}{x}\right)-b\cosh\left(a+\frac{b}{x}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^3,x]
[Out] (-(b*Cosh[a + b/x]) + x*Sinh[a + b/x])/(b^2*x)
```

fricas [A] time = 0.51, size = 34, normalized size = 1.17

$$-\frac{b\cosh\left(\frac{ax+b}{x}\right)-x\sinh\left(\frac{ax+b}{x}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="fricas")
[Out] -(b*cosh((a*x + b)/x) - x*sinh((a*x + b)/x))/(b^2*x)
```

giac [B] time = 0.24, size = 95, normalized size = 3.28

$$\frac{ae^{\left(\frac{ax+b}{x}\right)}+ae^{\left(-\frac{ax+b}{x}\right)}-\frac{(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x}-\frac{(ax+b)e^{\left(-\frac{ax+b}{x}\right)}}{x}+e^{\left(\frac{ax+b}{x}\right)}-e^{\left(-\frac{ax+b}{x}\right)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="giac")
[Out] 1/2*(a*e^((a*x + b)/x) + a*e^(-(a*x + b)/x) - (a*x + b)*e^((a*x + b)/x)/x -
 (a*x + b)*e^(-(a*x + b)/x)/x + e^((a*x + b)/x) - e^(-(a*x + b)/x))/b^2
```

maple [A] time = 0.02, size = 44, normalized size = 1.52

$$-\frac{\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)-a\cosh\left(a+\frac{b}{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^3,x)
```

```
[Out] -1/b^2*((a+b/x)*cosh(a+b/x)-sinh(a+b/x)-a*cosh(a+b/x))
```

maxima [C] time = 0.41, size = 48, normalized size = 1.66

$$-\frac{1}{4}b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x}\right)}{b^3}-\frac{e^{a}\Gamma\left(3,-\frac{b}{x}\right)}{b^3}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*(e^(-a)*gamma(3, b/x)/b^3 - e^a*gamma(3, -b/x)/b^3) - 1/2*sinh(a + b
/x)/x^2
```

mupad [B] time = 0.38, size = 29, normalized size = 1.00

$$\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)/x^3,x)
```

```
[Out] sinh(a + b/x)/b^2 - cosh(a + b/x)/(b*x)
```

sympy [A] time = 1.77, size = 29, normalized size = 1.00

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx}+\frac{\sinh\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0 \\ -\frac{\sinh(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**3,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x) + sinh(a + b/x)/b**2, Ne(b, 0)), (-sinh(a)/
(2*x**2), True))
```

3.35 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=46

$$-\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3}+\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[Out] -2*cosh(a+b/x)/b^3-cosh(a+b/x)/b/x^2+2*sinh(a+b/x)/b^2/x
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 3296, 2638}

$$\frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}-\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^4,x]
[Out] (-2*Cosh[a + b/x])/b^3 - Cosh[a + b/x]/(b*x^2) + (2*Sinh[a + b/x])/(b^2*x)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\,\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x} - \frac{2\,\text{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}$$
$$= -\frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.85

$$\frac{2bx\sinh\left(a+\frac{b}{x}\right)-\left(b^2+2x^2\right)\cosh\left(a+\frac{b}{x}\right)}{b^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^4,x]
[Out] (-((b^2 + 2*x^2)*Cosh[a + b/x]) + 2*b*x*Sinh[a + b/x])/(b^3*x^2)
```

fricas [A] time = 0.38, size = 43, normalized size = 0.93

$$\frac{2\,bx\sinh\left(\frac{ax+b}{x}\right)-\left(b^2+2\,x^2\right)\cosh\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="fricas")
[Out] (2*b*x*sinh((a*x + b)/x) - (b^2 + 2*x^2)*cosh((a*x + b)/x))/(b^3*x^2)
```

giac [B] time = 0.20, size = 214, normalized size = 4.65

$$-\frac{a^2e^{\left(\frac{ax+b}{x}\right)} + a^2e^{\left(-\frac{ax+b}{x}\right)} + 2\,ae^{\left(\frac{ax+b}{x}\right)} - \frac{2\,(ax+b)ae^{\left(\frac{ax+b}{x}\right)}}{x} - 2\,ae^{\left(-\frac{ax+b}{x}\right)} - \frac{2\,(ax+b)ae^{\left(-\frac{ax+b}{x}\right)}}{x} + \frac{(ax+b)^2e^{\left(\frac{ax+b}{x}\right)}}{x^2} - \frac{2\,(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x} +}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*e^((a*x + b)/x) + a^2*e^(-(a*x + b)/x) + 2*a*e^((a*x + b)/x) - 2*
(a*x + b)*a*e^((a*x + b)/x)/x - 2*a*e^(-(a*x + b)/x) - 2*(a*x + b)*a*e^(-(a
*x + b)/x)/x + (a*x + b)^2*e^((a*x + b)/x)/x^2 - 2*(a*x + b)*e^((a*x + b)/x
)/x + (a*x + b)^2*e^(-(a*x + b)/x)/x^2 + 2*(a*x + b)*e^(-(a*x + b)/x)/x + 2
*e^((a*x + b)/x) + 2*e^(-(a*x + b)/x))/b^3
```

maple [**B**] time = 0.02, size = 94, normalized size = 2.04

$$-\frac{\left(a+\frac{b}{x}\right)^2\cosh\left(a+\frac{b}{x}\right)-2\sinh\left(a+\frac{b}{x}\right)\left(a+\frac{b}{x}\right)+2\cosh\left(a+\frac{b}{x}\right)-2a\left(\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right)-\sinh\left(a+\frac{b}{x}\right)\right)+}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^4,x)
```

```
[Out] -1/b^3*((a+b/x)^2*cosh(a+b/x)-2*sinh(a+b/x)*(a+b/x)+2*cosh(a+b/x)-2*a*((a+b
/x)*cosh(a+b/x)-sinh(a+b/x))+a^2*cosh(a+b/x))
```

maxima [**C**] time = 0.35, size = 47, normalized size = 1.02

$$-\frac{1}{6}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x}\right)}{b^4}+\frac{e^{a}\Gamma\left(4,-\frac{b}{x}\right)}{b^4}\right)-\frac{\sinh\left(a+\frac{b}{x}\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(e^(-a)*gamma(4, b/x)/b^4 + e^a*gamma(4, -b/x)/b^4) - 1/3*sinh(a + b
/x)/x^3
```

mupad [**B**] time = 0.43, size = 67, normalized size = 1.46

$$-\frac{\mathrm{e}^{a+\frac{b}{x}}\,\left(\frac{1}{2\,b}-\frac{x}{b^2}+\frac{x^2}{b^3}\right)}{x^2}-\frac{\mathrm{e}^{-a-\frac{b}{x}}\,\left(\frac{x}{b^2}+\frac{1}{2\,b}+\frac{x^2}{b^3}\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)/x^4,x)
```

```
[Out] - (exp(a + b/x)*(1/(2*b) - x/b^2 + x^2/b^3))/x^2 - (exp(- a - b/x)*(x/b^2 +
 1/(2*b) + x^2/b^3))/x^2
```

sympy [A] time = 2.96, size = 46, normalized size = 1.00

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\sinh\left(a+\frac{b}{x}\right)}{b^2x} - \frac{2\cosh\left(a+\frac{b}{x}\right)}{b^3} & \text{for } b \neq 0 \\ -\frac{\sinh(a)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**4,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x**2) + 2*sinh(a + b/x)/(b**2*x) - 2*cosh(a +
b/x)/b**3, Ne(b, 0)), (-sinh(a)/(3*x**3), True))
```

3.36 $\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5} dx$

Optimal. Leaf size=62

$$\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}+\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}$$

```
[Out] -cosh(a+b/x)/b/x^3-6*cosh(a+b/x)/b^3/x+6*sinh(a+b/x)/b^4+3*sinh(a+b/x)/b^2/
x^2
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 3296, 2637}

$$\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}+\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}-\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x]/x^5,x]
```

```
[Out] -(Cosh[a + b/x]/(b*x^3)) - (6*Cosh[a + b/x])/(b^3*x) + (6*Sinh[a + b/x])/b^
4 + (3*Sinh[a + b/x])/(b^2*x^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x}\right)}{x^5}\,dx &= -\mathrm{Subst}\left(\int x^3\sinh(a+bx)\,dx, x, \frac{1}{x}\right) \\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} + \frac{3\,\mathrm{Subst}\left(\int x^2\cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b} \\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2} - \frac{6\,\mathrm{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2} \\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2} + \frac{6\,\mathrm{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x}\right)}{b^3} \\
&= -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3} - \frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x} + \frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4} + \frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.77

$$
\frac{3x\left(b^2+2x^2\right)\sinh\left(a+\frac{b}{x}\right)-b\left(b^2+6x^2\right)\cosh\left(a+\frac{b}{x}\right)}{b^4x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x]/x^5,x]
```

```
[Out] (-(b*(b^2 + 6*x^2)*Cosh[a + b/x]) + 3*x*(b^2 + 2*x^2)*Sinh[a + b/x])/(b^4*x
^3)
```

fricas [A] time = 0.38, size = 53, normalized size = 0.85

$$
-\frac{\left(b^3+6\,bx^2\right)\cosh\left(\frac{ax+b}{x}\right)-3\left(b^2x+2\,x^3\right)\sinh\left(\frac{ax+b}{x}\right)}{b^4x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] -((b^3 + 6*b*x^2)*cosh((a*x + b)/x) - 3*(b^2*x + 2*x^3)*sinh((a*x + b)/x))/
(b^4*x^3)
```

giac [B] time = 0.19, size = 386, normalized size = 6.23

$$a^{3}e^{\left(\frac{ax+b}{x}\right)} + a^{3}e^{\left(-\frac{ax+b}{x}\right)} + 3\,a^{2}e^{\left(\frac{ax+b}{x}\right)} - \frac{3\,(ax+b)a^{2}e^{\left(\frac{ax+b}{x}\right)}}{x} - 3\,a^{2}e^{\left(-\frac{ax+b}{x}\right)} - \frac{3\,(ax+b)a^{2}e^{\left(-\frac{ax+b}{x}\right)}}{x} + 6\,ae^{\left(\frac{ax+b}{x}\right)} + \frac{3\,(ax+b)^{2}ae^{\left(\frac{ax+b}{x}\right)}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*e^((a*x + b)/x) + a^3*e^(-(a*x + b)/x) + 3*a^2*e^((a*x + b)/x) - 3
*(a*x + b)*a^2*e^((a*x + b)/x)/x - 3*a^2*e^(-(a*x + b)/x) - 3*(a*x + b)*a^2
*e^(-(a*x + b)/x)/x + 6*a*e^((a*x + b)/x) + 3*(a*x + b)^2*a*e^((a*x + b)/x)
/x^2 - 6*(a*x + b)*a*e^((a*x + b)/x)/x + 6*a*e^(-(a*x + b)/x) + 3*(a*x + b)
^2*a*e^(-(a*x + b)/x)/x^2 + 6*(a*x + b)*a*e^(-(a*x + b)/x)/x - (a*x + b)^3*
e^((a*x + b)/x)/x^3 + 3*(a*x + b)^2*e^((a*x + b)/x)/x^2 - 6*(a*x + b)*e^((a
*x + b)/x)/x - (a*x + b)^3*e^(-(a*x + b)/x)/x^3 - 3*(a*x + b)^2*e^(-(a*x +
b)/x)/x^2 - 6*(a*x + b)*e^(-(a*x + b)/x)/x + 6*e^((a*x + b)/x) - 6*e^(-(a*x
 + b)/x))/b^4
```

maple [B] time = 0.02, size = 165, normalized size = 2.66

$$-\frac{\left(a+\frac{b}{x}\right)^{3}\cosh\left(a+\frac{b}{x}\right) - 3\sinh\left(a+\frac{b}{x}\right)\left(a+\frac{b}{x}\right)^{2} + 6\left(a+\frac{b}{x}\right)\cosh\left(a+\frac{b}{x}\right) - 6\sinh\left(a+\frac{b}{x}\right) - 3a\left(\left(a+\frac{b}{x}\right)^{2}\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x)/x^5,x)
```

```
[Out] -1/b^4*((a+b/x)^3*cosh(a+b/x)-3*sinh(a+b/x)*(a+b/x)^2+6*(a+b/x)*cosh(a+b/x)
-6*sinh(a+b/x)-3*a*((a+b/x)^2*cosh(a+b/x)-2*sinh(a+b/x)*(a+b/x)+2*cosh(a+b/
x))+3*a^2*((a+b/x)*cosh(a+b/x)-sinh(a+b/x))-a^3*cosh(a+b/x))
```

maxima [C] time = 0.39, size = 48, normalized size = 0.77

$$-\frac{1}{8}\,b\left(\frac{e^{(-a)}\Gamma\left(5,\frac{b}{x}\right)}{b^{5}} - \frac{e^{a}\Gamma\left(5,-\frac{b}{x}\right)}{b^{5}}\right) - \frac{\sinh\left(a+\frac{b}{x}\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*(e^(-a)*gamma(5, b/x)/b^5 - e^a*gamma(5, -b/x)/b^5) - 1/4*sinh(a + b
/x)/x^4
```

mupad [B] time = 0.44, size = 85, normalized size = 1.37

$$\frac{e^{a+\frac{b}{x}}\,\left(\frac{3\,x}{2\,b^2}-\frac{1}{2\,b}-\frac{3\,x^2}{b^3}+\frac{3\,x^3}{b^4}\right)}{x^3}-\frac{e^{-a-\frac{b}{x}}\,\left(\frac{3\,x}{2\,b^2}+\frac{1}{2\,b}+\frac{3\,x^2}{b^3}+\frac{3\,x^3}{b^4}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)/x^5,x)
```

```
[Out] (exp(a + b/x)*((3*x)/(2*b^2) - 1/(2*b) - (3*x^2)/b^3 + (3*x^3)/b^4))/x^3 -
(exp(- a - b/x)*((3*x)/(2*b^2) + 1/(2*b) + (3*x^2)/b^3 + (3*x^3)/b^4))/x^3
```

sympy [A] time = 4.90, size = 61, normalized size = 0.98

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x}\right)}{bx^3}+\frac{3\sinh\left(a+\frac{b}{x}\right)}{b^2x^2}-\frac{6\cosh\left(a+\frac{b}{x}\right)}{b^3x}+\frac{6\sinh\left(a+\frac{b}{x}\right)}{b^4} & \text{for}\ b\neq 0\\ -\frac{\sinh\left(a\right)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x)/x**5,x)
```

```
[Out] Piecewise((-cosh(a + b/x)/(b*x**3) + 3*sinh(a + b/x)/(b**2*x**2) - 6*cosh(a
 + b/x)/(b**3*x) + 6*sinh(a + b/x)/b**4, Ne(b, 0)), (-sinh(a)/(4*x**4), Tru
e))
```

3.37 $\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right)\,dx$

Optimal. Leaf size=146

$$-\frac{1}{8}e^{3a}b3^{m+1}\left(-\frac{b}{x}\right)^m(ex)^m\Gamma\left(-m-1,-\frac{3b}{x}\right)+\frac{3}{8}e^{a}b\left(-\frac{b}{x}\right)^m(ex)^m\Gamma\left(-m-1,-\frac{b}{x}\right)+\frac{3}{8}e^{-a}b\left(\frac{b}{x}\right)^m(ex)^m\Gamma\left(-m-1,\frac{b}{x}\right)-$$

```
[Out] -1/8*3^(1+m)*b*exp(3*a)*(-b/x)^m*(e*x)^m*GAMMA(-1-m,-3*b/x)+3/8*b*exp(a)*(-
b/x)^m*(e*x)^m*GAMMA(-1-m,-b/x)+3/8*b*(b/x)^m*(e*x)^m*GAMMA(-1-m,b/x)/exp(a
)-1/8*3^(1+m)*b*(b/x)^m*(e*x)^m*GAMMA(-1-m,3*b/x)/exp(3*a)
```

Rubi [A] time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5350, 3312, 3308, 2181}

$$-\frac{1}{8}e^{3a}b3^{m+1}\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{3b}{x}\right)+\frac{3}{8}e^{a}b\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{b}{x}\right)+\frac{3}{8}e^{-a}b\left(\frac{b}{x}\right)^m(ex)^m$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x]^3,x]
```

```
[Out] -(3^(1 + m)*b*E^(3*a)*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, (-3*b)/x])/8 + (3*b*
E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/8 + (3*b*(b/x)^m*(e*x)^m*Gamm
a[-1 - m, b/x])/(8*E^a) - (3^(1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (3*b)/
x])/(8*E^(3*a))
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x}\right) dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh^3(a+bx)\, dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(i\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int \left(\frac{3}{4} i x^{-2-m} \sinh(a+bx) - \frac{1}{4} i x^{-2-m} \sinh(3a+3bx)\right) dx, x\right.\right.$$
$$= -\left(\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh(3a+3bx)\, dx, x, \frac{1}{x}\right)\right) + \frac{1}{4}\left(3\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Su}$$
$$= -\left(\frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-i(3ia+3ibx)} x^{-2-m}\, dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int\right.$$
$$= -\frac{1}{8} 3^{1+m} b e^{3a} \left(-\frac{b}{x}\right)^m (ex)^m \Gamma\left(-1-m, -\frac{3b}{x}\right) + \frac{3}{8} b e^{a} \left(-\frac{b}{x}\right)^m (ex)^m \Gamma\left(-1-m, -\frac{b}{x}\right) + \frac{3}{8} b$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x]^3,x]
```

```
[Out] $Aborted
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax+b}{x}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x + b)/x)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(a + b/x)^3, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^3\left(a+\frac{b}{x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x)^3,x)
[Out] int((e*x)^m*sinh(a+b/x)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^3,x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(a + b/x)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+\frac{b}{x}\right)^3 (e\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)^3*(e*x)^m,x)
[Out] int(sinh(a + b/x)^3*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^3\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x)**3, x)
```

3.38 $\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=90

$$-e^{2a}b2^{m-1}\left(-\frac{b}{x}\right)^m(ex)^m\Gamma\left(-m-1,-\frac{2b}{x}\right)+e^{-2a}b2^{m-1}\left(\frac{b}{x}\right)^m(ex)^m\Gamma\left(-m-1,\frac{2b}{x}\right)-\frac{x(ex)^m}{2(m+1)}$$

```
[Out] -1/2*x*(e*x)^m/(1+m)-2^(-1+m)*b*exp(2*a)*(-b/x)^m*(e*x)^m*GAMMA(-1-m,-2*b/x
)+2^(-1+m)*b*(b/x)^m*(e*x)^m*GAMMA(-1-m,2*b/x)/exp(2*a)
```

Rubi [A] time = 0.16, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5350, 3312, 3307, 2181}

$$-e^{2a}b2^{m-1}\left(-\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,-\frac{2b}{x}\right)+e^{-2a}b2^{m-1}\left(\frac{b}{x}\right)^m(ex)^m\text{Gamma}\left(-m-1,\frac{2b}{x}\right)-\frac{x(ex)^m}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x]^2,x]
```

```
[Out] -(x*(e*x)^m)/(2*(1 + m)) - 2^(-1 + m)*b*E^(2*a)*(-(b/x))^m*(e*x)^m*Gamma[-1
 - m, (-2*b)/x] + (2^(-1 + m)*b*(b/x)^m*(e*x)^m*Gamma[-1 - m, (2*b)/x])/E^(
2*a)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^2\left(a+\frac{b}{x}\right)dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh^2(a+bx)\,dx,x,\frac{1}{x}\right)\right)\\ &= \left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int\left(\frac{x^{-2-m}}{2}-\frac{1}{2}x^{-2-m}\cosh(2a+2bx)\right)dx,x,\frac{1}{x}\right)\\ &= -\frac{x(ex)^m}{2(1+m)}-\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\cosh(2a+2bx)\,dx,x,\frac{1}{x}\right)\\ &= -\frac{x(ex)^m}{2(1+m)}-\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{-i(2ia+2ibx)}x^{-2-m}\,dx,x,\frac{1}{x}\right)-\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\\ &= -\frac{x(ex)^m}{2(1+m)}-2^{-1+m}be^{2a}\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{2b}{x}\right)+2^{-1+m}be^{-2a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1\right.\end{aligned}$$

Mathematica [A] time = 0.26, size = 88, normalized size = 0.98

$$-\frac{(ex)^m\left(b2^m(m+1)(\sinh(a)+\cosh(a))^2\left(-\frac{b}{x}\right)^m\Gamma\left(-m-1,-\frac{2b}{x}\right)-b2^m(m+1)(\cosh(a)-\sinh(a))^2\left(\frac{b}{x}\right)^m\Gamma\left(-m-\right.\right.}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x]^2,x]
```

```
[Out] -1/2*((e*x)^m*(x - 2^m*b*(1 + m)*(b/x)^m*Gamma[-1 - m, (2*b)/x]*(Cosh[a] -
Sinh[a])^2 + 2^m*b*(1 + m)*(-(b/x))^m*Gamma[-1 - m, (-2*b)/x]*(Cosh[a] + Si
nh[a])^2))/(1 + m)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m\sinh\left(\frac{ax+b}{x}\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x + b)/x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a + \frac{b}{x}\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x)^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^2\left(a + \frac{b}{x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x)^2,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} e^m \int e^{\left(m \log(x) + 2a + \frac{2b}{x}\right)} \, dx + \frac{1}{4} e^m \int e^{\left(m \log(x) - 2a - \frac{2b}{x}\right)} \, dx - \frac{(ex)^{m+1}}{2e(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*e^m*integrate(e^(m*log(x) + 2*a + 2*b/x), x) + 1/4*e^m*integrate(e^(m*l
og(x) - 2*a - 2*b/x), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a + \frac{b}{x}\right)^2 (e\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)^2*(e*x)^m,x)
```

```
[Out] int(sinh(a + b/x)^2*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^2\left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x)**2, x)
```

3.39 $\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=67

$$-\frac{1}{2}e^a b\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,-\frac{b}{x}\right)-\frac{1}{2}e^{-a}b\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-m-1,\frac{b}{x}\right)$$

```
[Out] -1/2*b*exp(a)*(-b/x)^m*(e*x)^m*GAMMA(-1-m,-b/x)-1/2*b*(b/x)^m*(e*x)^m*GAMMA
(-1-m,b/x)/exp(a)
```

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5350, 3308, 2181}

$$-\frac{1}{2}e^a b\left(-\frac{b}{x}\right)^m (ex)^m\text{Gamma}\left(-m-1,-\frac{b}{x}\right)-\frac{1}{2}e^{-a}b\left(\frac{b}{x}\right)^m (ex)^m\text{Gamma}\left(-m-1,\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x],x]
```

```
[Out] -(b*E^a*(-(b/x))^m*(e*x)^m*Gamma[-1 - m, -(b/x)])/2 - (b*(b/x)^m*(e*x)^m*Ga
mma[-1 - m, b/x])/(2*E^a)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh(a+bx)\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{-i(ia+ibx)}x^{-2-m}\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{i}\right.$$
$$= -\frac{1}{2}be^a\left(-\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,-\frac{b}{x}\right) - \frac{1}{2}be^{-a}\left(\frac{b}{x}\right)^m (ex)^m\Gamma\left(-1-m,\frac{b}{x}\right)$$

Mathematica [A] time = 0.09, size = 63, normalized size = 0.94

$$-\frac{1}{2}b(ex)^m\left((\sinh(a)+\cosh(a))\left(-\frac{b}{x}\right)^m\Gamma\left(-m-1,-\frac{b}{x}\right)+(\cosh(a)-\sinh(a))\left(\frac{b}{x}\right)^m\Gamma\left(-m-1,\frac{b}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x],x]
[Out] -1/2*(b*(e*x)^m*((b/x)^m*Gamma[-1 - m, b/x]*(Cosh[a] - Sinh[a]) + (-(b/x))^
m*Gamma[-1 - m, -(b/x)]*(Cosh[a] + Sinh[a])))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m\sinh\left(\frac{ax+b}{x}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="fricas")
[Out] integral((e*x)^m*sinh((a*x + b)/x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\sinh\left(a+\frac{b}{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(a + b/x), x)
```

maple [C] time = 0.11, size = 70, normalized size = 1.04

$$\frac{(ex)^m\, b\,\mathrm{hypergeom}\left(\left[-\frac{m}{2}\right],\left[\frac{3}{2},1-\frac{m}{2}\right],\frac{b^2}{4x^2}\right)\cosh(a)}{m}+\frac{(ex)^m\, x\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}-\frac{m}{2}\right],\left[\frac{1}{2},\frac{1}{2}-\frac{m}{2}\right],\frac{b^2}{4x^2}\right)\sinh(a)}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x),x)
```

```
[Out] (e*x)^m*b/m*hypergeom([-1/2*m],[3/2,1-1/2*m],1/4/x^2*b^2)*cosh(a)+(e*x)^m/(
1+m)*x*hypergeom([-1/2-1/2*m],[1/2,1/2-1/2*m],1/4/x^2*b^2)*sinh(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+\frac{b}{x}\right)\, (e\, x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x)*(e*x)^m,x)
```

```
[Out] int(sinh(a + b/x)*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x), x)
```

3.40 $\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=26

$$x^{-m}(ex)^m \mathrm{Int}\left(x^m \mathrm{csch}\left(a+\frac{b}{x}\right),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(a+b/x),x)/(x^m)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b/x],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b/x], x])/x^m
```

Rubi steps

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x}\right) dx = \left(x^{-m}(ex)^m\right)\int x^m \mathrm{csch}\left(a+\frac{b}{x}\right) dx$$

Mathematica [A] time = 3.42, size = 0, normalized size = 0.00

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x}\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b/x],x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b/x], x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex)^m}{\sinh\left(\frac{ax+b}{x}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="fricas")
[Out] integral((e*x)^m/sinh((a*x + b)/x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="giac")
[Out] integrate((e*x)^m/sinh(a + b/x), x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a+b/x),x)
[Out] int((e*x)^m/sinh(a+b/x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a+\frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x),x, algorithm="maxima")
[Out] integrate((e*x)^m/sinh(a + b/x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{\sinh\left(a+\frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a + b/x),x)
```

```
[Out] int((e*x)^m/sinh(a + b/x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a + \frac{b}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(a+b/x),x)
```

```
[Out] Integral((e*x)**m/sinh(a + b/x), x)
```

3.41 $\int x^4 \sinh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=104

$$-\frac{2}{15}\sqrt{\pi}\,e^{-a}b^{5/2}\text{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{2}{15}\sqrt{\pi}\,e^{a}b^{5/2}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right)$$

```
[Out] 2/15*b*x^3*cosh(a+b/x^2)+4/15*b^2*x*sinh(a+b/x^2)+1/5*x^5*sinh(a+b/x^2)-2/1
5*b^(5/2)*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)-2/15*b^(5/2)*exp(a)*erfi(b^(1/2)/x
)*Pi^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5346, 5326, 5327, 5299, 2204, 2205}

$$-\frac{2}{15}\sqrt{\pi}\,e^{-a}b^{5/2}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{2}{15}\sqrt{\pi}\,e^{a}b^{5/2}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{4}{15}b^2x\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{15}bx^3\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x^3*Cosh[a + b/x^2])/15 - (2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(15*E^a)
 - (2*b^(5/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/15 + (4*b^2*x*Sinh[a + b/x^2])/
15 + (x^5*Sinh[a + b/x^2])/5
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5327

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*
x)^(m + 1)*Cosh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^4 \sinh\left(a+\frac{b}{x^2}\right) dx = -\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^6} dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{\cosh\left(a+bx^2\right)}{x^4} dx, x, \frac{1}{x}\right)$$
$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}\left(4b^2\right)\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^2} dx, x, \frac{1}{x}\right.$$
$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) + \frac{4}{15}b^2x \sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}\left(8b^3\right)\text{Subst}\left(\int\right.$$
$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) + \frac{4}{15}b^2x \sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{5}x^5 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{15}\left(4b^3\right)\text{Subst}\left(\int\right.$$
$$= \frac{2}{15}bx^3 \cosh\left(a+\frac{b}{x^2}\right) - \frac{2}{15}b^{5/2}e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{2}{15}b^{5/2}e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{4}{15}b^2x \sinh\left(\right.$$

Mathematica [A] time = 0.11, size = 102, normalized size = 0.98

$$\frac{1}{15}\left(2\sqrt{\pi}\,b^{5/2}(\sinh(a)-\cosh(a))\text{erf}\left(\frac{\sqrt{b}}{x}\right) - 2\sqrt{\pi}\,b^{5/2}(\sinh(a)+\cosh(a))\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + 4b^2x \sinh\left(a+\frac{b}{x^2}\right) + 3x^5 \text{s}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x^3*Cosh[a + b/x^2] + 2*b^(5/2)*Sqrt[Pi]*Erf[Sqrt[b]/x]*(-Cosh[a] + Si
nh[a]) - 2*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) + 4*b^2*x*S
inh[a + b/x^2] + 3*x^5*Sinh[a + b/x^2])/15
```

fricas [B] time = 0.44, size = 323, normalized size = 3.11

$$-\frac{3\,x^5 - 2\,bx^3 + 4\,b^2x - \left(3\,x^5 + 2\,bx^3 + 4\,b^2x\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2 - 4\,\sqrt{\pi}\left(b^2\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right) + b^2\cosh\left(\frac{ax^2+b}{x^2}\right)s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/30*(3*x^5 - 2*b*x^3 + 4*b^2*x - (3*x^5 + 2*b*x^3 + 4*b^2*x)*cosh((a*x^2
+ b)/x^2)^2 - 4*sqrt(pi)*(b^2*cosh(a)*cosh((a*x^2 + b)/x^2) + b^2*cosh((a*x
^2 + b)/x^2)*sinh(a) + (b^2*cosh(a) + b^2*sinh(a))*sinh((a*x^2 + b)/x^2))*s
qrt(-b)*erf(sqrt(-b)/x) + 4*sqrt(pi)*(b^2*cosh(a)*cosh((a*x^2 + b)/x^2) - b
^2*cosh((a*x^2 + b)/x^2)*sinh(a) + (b^2*cosh(a) - b^2*sinh(a))*sinh((a*x^2
+ b)/x^2))*sqrt(b)*erf(sqrt(b)/x) - 2*(3*x^5 + 2*b*x^3 + 4*b^2*x)*cosh((a*x
^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) - (3*x^5 + 2*b*x^3 + 4*b^2*x)*sinh((a*x^
2 + b)/x^2)^2)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*sinh(a + b/x^2), x)
```

maple [A] time = 0.08, size = 138, normalized size = 1.33

$$-\frac{e^{-a}x^5e^{-\frac{b}{x^2}}}{10} + \frac{e^{-a}b\,x^3e^{-\frac{b}{x^2}}}{15} - \frac{2\,e^{-a}b^{\frac{5}{2}}\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{15} - \frac{2\,e^{-a}e^{-\frac{b}{x^2}}b^2x}{15} + \frac{e^{a}x^5e^{\frac{b}{x^2}}}{10} + \frac{e^{a}b\,x^3e^{\frac{b}{x^2}}}{15} + \frac{2\,e^{a}b^2e^{\frac{b}{x^2}}x}{15} - \frac{2\,e^{a}b^3\sqrt{\pi}\ e}{15\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sinh(a+b/x^2),x)
```

```
[Out] -1/10*exp(-a)*x^5*exp(-b/x^2)+1/15*exp(-a)*b*x^3*exp(-b/x^2)-2/15*exp(-a)*b
^(5/2)*Pi^(1/2)*erf(b^(1/2)/x)-2/15*exp(-a)*exp(-b/x^2)*b^2*x+1/10*exp(a)*x
^5*exp(b/x^2)+1/15*exp(a)*b*x^3*exp(b/x^2)+2/15*exp(a)*b^2*exp(b/x^2)*x-2/1
5*exp(a)*b^3*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

maxima [A] time = 1.34, size = 62, normalized size = 0.60

$$\frac{1}{5}x^5\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{10}\left(x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}e^{(-a)}\Gamma\left(-\frac{3}{2},\frac{b}{x^2}\right)+x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}e^{a}\Gamma\left(-\frac{3}{2},-\frac{b}{x^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*sinh(a + b/x^2) + 1/10*(x^3*(b/x^2)^(3/2)*e^(-a)*gamma(-3/2, b/x^2)
 + x^3*(-b/x^2)^(3/2)*e^a*gamma(-3/2, -b/x^2))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\sinh\left(a+\frac{b}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sinh(a + b/x^2),x)
```

```
[Out] int(x^4*sinh(a + b/x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**4*sinh(a + b/x**2), x)
```

3.42 $\int x^3 \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=62

$$-\frac{1}{4}b^2\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] 1/4*b*x^2*cosh(a+b/x^2)-1/4*b^2*cosh(a)*Shi(b/x^2)-1/4*b^2*Chi(b/x^2)*sinh(
a)+1/4*x^4*sinh(a+b/x^2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{4}b^2\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{4}b^2\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sinh[a + b/x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b/x^2])/4 - (b^2*CoshIntegral[b/x^2]*Sinh[a])/4 + (x^4*Sinh
[a + b/x^2])/4 - (b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x^3 \sinh\left(a+\frac{b}{x^2}\right) dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b\operatorname{Subst}\left(\int \frac{\cosh(a+bx)}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\operatorname{Subst}\left(\int \frac{\sinh(a+bx)}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}\left(b^2\cosh(a)\right)\operatorname{Subst}\left(\int \frac{\sinh(bx)}{x}\,dx, x, \frac{1}{x^2}\right.\\
&= \frac{1}{4}bx^2\cosh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\operatorname{Chi}\left(\frac{b}{x^2}\right)\sinh(a) + \frac{1}{4}x^4\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{4}b^2\cosh(a)\operatorname{Shi}\left(\frac{b}{x^2}\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.90

$$\frac{1}{4}\left(-b^2\sinh(a)\operatorname{Chi}\left(\frac{b}{x^2}\right) - b^2\cosh(a)\operatorname{Shi}\left(\frac{b}{x^2}\right) + bx^2\cosh\left(a+\frac{b}{x^2}\right) + x^4\sinh\left(a+\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sinh[a + b/x^2],x]
```

```
[Out] (b*x^2*Cosh[a + b/x^2] - b^2*CoshIntegral[b/x^2]*Sinh[a] + x^4*Sinh[a + b/x
^2] - b^2*Cosh[a]*SinhIntegral[b/x^2])/4
```

fricas [A] time = 0.43, size = 89, normalized size = 1.44

$$\frac{1}{4}x^4\sinh\left(\frac{ax^2+b}{x^2}\right) + \frac{1}{4}bx^2\cosh\left(\frac{ax^2+b}{x^2}\right) - \frac{1}{8}\left(b^2\operatorname{Ei}\left(\frac{b}{x^2}\right) - b^2\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a) - \frac{1}{8}\left(b^2\operatorname{Ei}\left(\frac{b}{x^2}\right) + b^2\operatorname{Ei}\left(-\frac{b}{x^2}\right)\right)\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*sinh((a*x^2 + b)/x^2) + 1/4*b*x^2*cosh((a*x^2 + b)/x^2) - 1/8*(b^2*
Ei(b/x^2) - b^2*Ei(-b/x^2))*cosh(a) - 1/8*(b^2*Ei(b/x^2) + b^2*Ei(-b/x^2))*
sinh(a)
```

giac [B] time = 0.20, size = 353, normalized size = 5.69

$$\frac{a^2b^3\mathrm{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)} - a^2b^3\mathrm{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^{a} - \frac{2\left(ax^2+b\right)ab^3\mathrm{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)}}{x^2} + \frac{2\left(ax^2+b\right)ab^3\mathrm{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^{a}}{x^2} - ab^3e^{\left(\frac{ax^2+b}{x^2}\right.}}{8\left(a^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2*b^3*Ei(a - (a*x^2 + b)/x^2)*e^(-a) - a^2*b^3*Ei(-a + (a*x^2 + b)/x
^2)*e^a - 2*(a*x^2 + b)*a*b^3*Ei(a - (a*x^2 + b)/x^2)*e^(-a)/x^2 + 2*(a*x^2
 + b)*a*b^3*Ei(-a + (a*x^2 + b)/x^2)*e^a/x^2 - a*b^3*e^((a*x^2 + b)/x^2) -
a*b^3*e^(-(a*x^2 + b)/x^2) + b^3*e^((a*x^2 + b)/x^2) - b^3*e^(-(a*x^2 + b)/
x^2) + (a*x^2 + b)^2*b^3*Ei(a - (a*x^2 + b)/x^2)*e^(-a)/x^4 - (a*x^2 + b)^2
*b^3*Ei(-a + (a*x^2 + b)/x^2)*e^a/x^4 + (a*x^2 + b)*b^3*e^((a*x^2 + b)/x^2)
/x^2 + (a*x^2 + b)*b^3*e^(-(a*x^2 + b)/x^2)/x^2)/((a^2 - 2*(a*x^2 + b)*a/x^
2 + (a*x^2 + b)^2/x^4)*b)
```

maple [A] time = 0.04, size = 93, normalized size = 1.50

$$-\frac{e^{-a}x^4e^{-\frac{b}{x^2}}}{8} + \frac{e^{-a}b\,x^2e^{-\frac{b}{x^2}}}{8} - \frac{e^{-a}b^2\,\mathrm{Ei}\left(1,\frac{b}{x^2}\right)}{8} + \frac{e^{a}x^4e^{\frac{b}{x^2}}}{8} + \frac{e^{a}b\,e^{\frac{b}{x^2}}x^2}{8} + \frac{e^{a}b^2\,\mathrm{Ei}\left(1,-\frac{b}{x^2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a+b/x^2),x)
```

```
[Out] -1/8*exp(-a)*x^4*exp(-b/x^2)+1/8*exp(-a)*b*x^2*exp(-b/x^2)-1/8*exp(-a)*b^2*
Ei(1,b/x^2)+1/8*exp(a)*x^4*exp(b/x^2)+1/8*exp(a)*b*exp(b/x^2)*x^2+1/8*exp(a
)*b^2*Ei(1,-b/x^2)
```

maxima [A] time = 0.51, size = 44, normalized size = 0.71

$$\frac{1}{4}\,x^4\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{8}\left(be^{(-a)}\Gamma\left(-1,\frac{b}{x^2}\right) - be^{a}\Gamma\left(-1,-\frac{b}{x^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*sinh(a + b/x^2) + 1/8*(b*e^(-a)*gamma(-1, b/x^2) - b*e^a*gamma(-1,
-b/x^2))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sinh(a + b/x^2),x)
```

```
[Out] int(x^3*sinh(a + b/x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**3*sinh(a + b/x**2), x)
```

3.43 $\int x^2 \sinh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=86

$$\frac{1}{3}\sqrt{\pi}\,e^{-a}b^{3/2}\text{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{3}\sqrt{\pi}\,e^{a}b^{3/2}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] 2/3*b*x*cosh(a+b/x^2)+1/3*x^3*sinh(a+b/x^2)+1/3*b^(3/2)*erf(b^(1/2)/x)*Pi^(
1/2)/exp(a)-1/3*b^(3/2)*exp(a)*erfi(b^(1/2)/x)*Pi^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5346, 5326, 5327, 5298, 2204, 2205}

$$\frac{1}{3}\sqrt{\pi}\,e^{-a}b^{3/2}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{3}\sqrt{\pi}\,e^{a}b^{3/2}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)+\frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right)+\frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x*Cosh[a + b/x^2])/3 + (b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x])/(3*E^a) - (b^
(3/2)*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/3 + (x^3*Sinh[a + b/x^2])/3
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5327

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*
x)^(m + 1)*Cosh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sinh\left(a+\frac{b}{x^2}\right) dx &= -\text{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^4} dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3 \sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{\cosh\left(a+bx^2\right)}{x^2} dx, x, \frac{1}{x}\right)\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right) - \frac{1}{3}\left(4b^2\right)\text{Subst}\left(\int \sinh\left(a+bx^2\right) dx, x, \frac{1}{x}\right)\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}\left(2b^2\right)\text{Subst}\left(\int e^{-a-bx^2} dx, x, \frac{1}{x}\right) - \frac{1}{3}\left(2b^2\right.\\
&= \frac{2}{3}bx\cosh\left(a+\frac{b}{x^2}\right) + \frac{1}{3}b^{3/2}e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{3}b^{3/2}e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + \frac{1}{3}x^3\sinh\left(a+\frac{b}{x^2}\right.
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 84, normalized size = 0.98

$$
\frac{1}{3}\left(\sqrt{\pi}\,b^{3/2}(\cosh(a)-\sinh(a))\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \sqrt{\pi}\,b^{3/2}(\sinh(a)+\cosh(a))\text{erfi}\left(\frac{\sqrt{b}}{x}\right) + 2bx\cosh\left(a+\frac{b}{x^2}\right) + x^3\sinh\left(\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b/x^2],x]
```

```
[Out] (2*b*x*Cosh[a + b/x^2] + b^(3/2)*Sqrt[Pi]*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]
) - b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) + x^3*Sinh[a + b/x
^2])/3
```

fricas [B] time = 0.43, size = 267, normalized size = 3.10

$$-\frac{x^3 - \left(x^3 + 2\,bx\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2 - 2\,\sqrt{\pi}\left(b\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right) + b\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a) + (b\cosh(a) + b\sinh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3 - (x^3 + 2*b*x)*cosh((a*x^2 + b)/x^2)^2 - 2*sqrt(pi)*(b*cosh(a)*c
osh((a*x^2 + b)/x^2) + b*cosh((a*x^2 + b)/x^2)*sinh(a) + (b*cosh(a) + b*sin
h(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) - 2*sqrt(pi)*(b*cosh(
a)*cosh((a*x^2 + b)/x^2) - b*cosh((a*x^2 + b)/x^2)*sinh(a) + (b*cosh(a) - b
*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) - 2*(x^3 + 2*b*x)*c
osh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) - (x^3 + 2*b*x)*sinh((a*x^2 + b)
/x^2)^2 - 2*b*x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(a + b/x^2), x)
```

maple [A] time = 0.05, size = 103, normalized size = 1.20

$$-\frac{\mathrm{e}^{-a}x^3\mathrm{e}^{-\frac{b}{x^2}}}{6} + \frac{\mathrm{e}^{-a}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)b^{\frac{3}{2}}\sqrt{\pi}}{3} + \frac{\mathrm{e}^{-a}\mathrm{e}^{-\frac{b}{x^2}}bx}{3} + \frac{\mathrm{e}^{a}x^3\mathrm{e}^{\frac{b}{x^2}}}{6} + \frac{\mathrm{e}^{a}b\,\mathrm{e}^{\frac{b}{x^2}}x}{3} - \frac{\mathrm{e}^{a}b^2\sqrt{\pi}\;\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b/x^2),x)
```

```
[Out] -1/6*exp(-a)*x^3*exp(-b/x^2)+1/3*exp(-a)*erf(b^(1/2)/x)*b^(3/2)*Pi^(1/2)+1/
3*exp(-a)*exp(-b/x^2)*b*x+1/6*exp(a)*x^3*exp(b/x^2)+1/3*exp(a)*b*exp(b/x^2)
*x-1/3*exp(a)*b^2*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

maxima [A] time = 0.39, size = 58, normalized size = 0.67

$$\frac{1}{3}\,x^3\sinh\left(a+\frac{b}{x^2}\right)+\frac{1}{6}\left(x\sqrt{\frac{b}{x^2}}\,e^{(-a)}\Gamma\left(-\frac{1}{2},\frac{b}{x^2}\right)+x\sqrt{-\frac{b}{x^2}}\,e^{a}\Gamma\left(-\frac{1}{2},-\frac{b}{x^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh(a + b/x^2) + 1/6*(x*sqrt(b/x^2)*e^(-a)*gamma(-1/2, b/x^2) + x*
sqrt(-b/x^2)*e^a*gamma(-1/2, -b/x^2))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\sinh\left(a+\frac{b}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b/x^2),x)
```

```
[Out] int(x^2*sinh(a + b/x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b/x**2),x)
```

```
[Out] Integral(x**2*sinh(a + b/x**2), x)
```

3.44 $\int x \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=42

$$-\frac{1}{2}b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] -1/2*b*Chi(b/x^2)*cosh(a)-1/2*b*Shi(b/x^2)*sinh(a)+1/2*x^2*sinh(a+b/x^2)
```

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5320, 3297, 3303, 3298, 3301}

$$-\frac{1}{2}b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b/x^2],x]
```

```
[Out] -(b*Cosh[a]*CoshIntegral[b/x^2])/2 + (x^2*Sinh[a + b/x^2])/2 - (b*Sinh[a]*S
inhIntegral[b/x^2])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\sinh\left(a+\frac{b}{x^2}\right)dx &= -\left(\frac{1}{2}\text{Subst}\left(\int\frac{\sinh(a+bx)}{x^2}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{2}b\,\text{Subst}\left(\int\frac{\cosh(a+bx)}{x}\,dx,x,\frac{1}{x^2}\right)\\ &= \frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{2}(b\cosh(a))\,\text{Subst}\left(\int\frac{\cosh(bx)}{x}\,dx,x,\frac{1}{x^2}\right)-\frac{1}{2}(b\sinh(a))\,\text{Subst}\left(\right.\\ &= -\frac{1}{2}b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)+\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{2}b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 0.93

$$\frac{1}{2}\left(-b\cosh(a)\text{Chi}\left(\frac{b}{x^2}\right)-b\sinh(a)\text{Shi}\left(\frac{b}{x^2}\right)+x^2\sinh\left(a+\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b/x^2],x]
[Out] (-(b*Cosh[a]*CoshIntegral[b/x^2]) + x^2*Sinh[a + b/x^2] - b*Sinh[a]*SinhInt
egral[b/x^2])/2
```

fricas [A] time = 0.47, size = 63, normalized size = 1.50

$$\frac{1}{2}x^2\sinh\left(\frac{ax^2+b}{x^2}\right)-\frac{1}{4}\left(b\text{Ei}\left(\frac{b}{x^2}\right)+b\text{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a)-\frac{1}{4}\left(b\text{Ei}\left(\frac{b}{x^2}\right)-b\text{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*sinh((a*x^2 + b)/x^2) - 1/4*(b*Ei(b/x^2) + b*Ei(-b/x^2))*cosh(a) -
1/4*(b*Ei(b/x^2) - b*Ei(-b/x^2))*sinh(a)
```

giac [B] time = 0.18, size = 193, normalized size = 4.60

$$-\frac{ab^2\mathrm{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)}-\frac{(ax^2+b)b^2\mathrm{Ei}\left(a-\frac{ax^2+b}{x^2}\right)e^{(-a)}}{x^2}-b^2e^{\left(-\frac{ax^2+b}{x^2}\right)}}{4\left(a-\frac{ax^2+b}{x^2}\right)b}-\frac{ab^2\mathrm{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^{a}-\frac{(ax^2+b)b^2\mathrm{Ei}\left(-a+\frac{ax^2+b}{x^2}\right)e^{a}}{x^2}+b^2}{4\left(a-\frac{ax^2+b}{x^2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] -1/4*(a*b^2*Ei(a - (a*x^2 + b)/x^2)*e^(-a) - (a*x^2 + b)*b^2*Ei(a - (a*x^2
+ b)/x^2)*e^(-a)/x^2 - b^2*e^(-(a*x^2 + b)/x^2))/((a - (a*x^2 + b)/x^2)*b)
- 1/4*(a*b^2*Ei(-a + (a*x^2 + b)/x^2)*e^a - (a*x^2 + b)*b^2*Ei(-a + (a*x^2
+ b)/x^2)*e^a/x^2 + b^2*e^((a*x^2 + b)/x^2))/((a - (a*x^2 + b)/x^2)*b)
```

maple [A] time = 0.02, size = 58, normalized size = 1.38

$$-\frac{e^{-a}x^2e^{-\frac{b}{x^2}}}{4}+\frac{e^{-a}b\,\mathrm{Ei}\left(1,\frac{b}{x^2}\right)}{4}+\frac{e^{a}e^{\frac{b}{x^2}}x^2}{4}+\frac{e^{a}b\,\mathrm{Ei}\left(1,-\frac{b}{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b/x^2),x)
```

```
[Out] -1/4*exp(-a)*x^2*exp(-b/x^2)+1/4*exp(-a)*b*Ei(1,b/x^2)+1/4*exp(a)*exp(b/x^2
)*x^2+1/4*exp(a)*b*Ei(1,-b/x^2)
```

maxima [A] time = 0.39, size = 39, normalized size = 0.93

$$\frac{1}{2}x^2\sinh\left(a+\frac{b}{x^2}\right)-\frac{1}{4}\left(\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)}+\mathrm{Ei}\left(\frac{b}{x^2}\right)e^{a}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh(a + b/x^2) - 1/4*(Ei(-b/x^2)*e^(-a) + Ei(b/x^2)*e^a)*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sinh\left(a+\frac{b}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b/x^2),x)
```

```
[Out] int(x*sinh(a + b/x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b/x**2),x)
```

```
[Out] Integral(x*sinh(a + b/x**2), x)
```

3.45 $\int \sinh\left(a+\frac{b}{x^2}\right) dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

```
[Out] x*sinh(a+b/x^2)-1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)-1/2*exp(a)*erfi(
b^(1/2)/x)*b^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5302, 5326, 5299, 2204, 2205}

$$-\frac{1}{2}\sqrt{\pi}\,e^{-a}\sqrt{b}\,\mathrm{Erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{\pi}\,e^{a}\sqrt{b}\,\mathrm{Erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2],x]
```

```
[Out] -(Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x])/(2*E^a) - (Sqrt[b]*E^a*Sqrt[Pi]*Erfi[Sqr
t[b]/x])/2 + x*Sinh[a + b/x^2]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5302

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5326

```
Int[((e_.)*(x_))^(m_)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[((e*
x)^(m + 1)*Sinh[c + d*x^n])/(e*(m + 1)), x] - Dist[(d*n)/(e^n*(m + 1)), Int
[(e*x)^(m + n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
 && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sinh\left(a+\frac{b}{x^2}\right)\,dx &= -\operatorname{Subst}\left(\int \frac{\sinh\left(a+bx^2\right)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= x\sinh\left(a+\frac{b}{x^2}\right)-(2b)\operatorname{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= x\sinh\left(a+\frac{b}{x^2}\right)-b\operatorname{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)-b\operatorname{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2}\sqrt{b}\,e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)-\frac{1}{2}\sqrt{b}\,e^{a}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)+x\sinh\left(a+\frac{b}{x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 70, normalized size = 1.04

$$
-\frac{1}{2}\sqrt{\pi}\sqrt{b}\left((\cosh(a)-\sinh(a))\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)+(\sinh(a)+\cosh(a))\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)\right)+x\sinh(a)\cosh\left(\frac{b}{x^2}\right)+x\cosh(a)\sinh
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2],x]
```

```
[Out] x*Cosh[b/x^2]*Sinh[a] - (Sqrt[b]*Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a
]) + Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a])))/2 + x*Cosh[a]*Sinh[b/x^2]
```

fricas [B] time = 0.56, size = 228, normalized size = 3.40

$$
x\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(\cosh(a)+\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)\sqrt{-b}\operatorname{erf}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) +
cosh((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) + sinh(a))*sinh((a*x^2 + b)/x^2))*
sqrt(-b)*erf(sqrt(-b)/x) - sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) - cosh((
a*x^2 + b)/x^2)*sinh(a) + (cosh(a) - sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b
)*erf(sqrt(b)/x) + 2*x*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + x*sinh
((a*x^2 + b)/x^2)^2 - x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2), x)
```

maple [A] time = 0.05, size = 70, normalized size = 1.04

$$-\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\mathrm{e}^{-a}\sqrt{b}\,\sqrt{\pi}}{2}-\frac{\mathrm{e}^{-a}\mathrm{e}^{-\frac{b}{x^2}}x}{2}+\frac{\mathrm{e}^{a}\mathrm{e}^{\frac{b}{x^2}}x}{2}-\frac{\mathrm{e}^{a}b\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2),x)
```

```
[Out] -1/2*erf(b^(1/2)/x)*exp(-a)*b^(1/2)*Pi^(1/2)-1/2*exp(-a)*exp(-b/x^2)*x+1/2*
exp(a)*exp(b/x^2)*x-1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

maxima [A] time = 0.36, size = 71, normalized size = 1.06

$$-\frac{1}{2}b\left(\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\frac{b}{x^2}}\right)-1\right)e^{(-a)}}{x\sqrt{\frac{b}{x^2}}}+\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-\frac{b}{x^2}}\right)-1\right)e^{a}}{x\sqrt{-\frac{b}{x^2}}}\right)+x\sinh\left(a+\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(sqrt(pi)*(erf(sqrt(b/x^2)) - 1)*e^(-a)/(x*sqrt(b/x^2)) + sqrt(pi)*(
erf(sqrt(-b/x^2)) - 1)*e^a/(x*sqrt(-b/x^2))) + x*sinh(a + b/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2),x)
```

```
[Out] int(sinh(a + b/x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2),x)
```

```
[Out] Integral(sinh(a + b/x**2), x)
```

3.46 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*cosh(a)*Shi(b/x^2)-1/2*Chi(b/x^2)*sinh(a)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5318, 5317, 5316}

$$-\frac{1}{2}\sinh(a)\text{Chi}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cosh(a)\text{Shi}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x,x]
```

```
[Out] -(CoshIntegral[b/x^2]*Sinh[a])/2 - (Cosh[a]*SinhIntegral[b/x^2])/2
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx = \cosh(a)\int \frac{\sinh\left(\frac{b}{x^2}\right)}{x}\,dx + \sinh(a)\int \frac{\cosh\left(\frac{b}{x^2}\right)}{x}\,dx$$
$$= -\frac{1}{2}\mathrm{Chi}\left(\frac{b}{x^2}\right)\sinh(a) - \frac{1}{2}\cosh(a)\mathrm{Shi}\left(\frac{b}{x^2}\right)$$

Mathematica [**A**] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{2}\left(\sinh(a)\left(-\mathrm{Chi}\left(\frac{b}{x^2}\right)\right) - \cosh(a)\mathrm{Shi}\left(\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x,x]
[Out] (-(CoshIntegral[b/x^2]*Sinh[a]) - Cosh[a]*SinhIntegral[b/x^2])/2
```

fricas [**A**] time = 0.43, size = 39, normalized size = 1.56

$$-\frac{1}{4}\left(\mathrm{Ei}\left(\frac{b}{x^2}\right) - \mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\cosh(a) - \frac{1}{4}\left(\mathrm{Ei}\left(\frac{b}{x^2}\right) + \mathrm{Ei}\left(-\frac{b}{x^2}\right)\right)\sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="fricas")
[Out] -1/4*(Ei(b/x^2) - Ei(-b/x^2))*cosh(a) - 1/4*(Ei(b/x^2) + Ei(-b/x^2))*sinh(a
)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="giac")
[Out] integrate(sinh(a + b/x^2)/x, x)
```

maple [**A**] time = 0.03, size = 27, normalized size = 1.08

$$-\frac{\mathrm{e}^{-a}\,\mathrm{Ei}\left(1,\frac{b}{x^2}\right)}{4} + \frac{\mathrm{e}^{a}\,\mathrm{Ei}\left(1,-\frac{b}{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x,x)
```

```
[Out] -1/4*exp(-a)*Ei(1,b/x^2)+1/4*exp(a)*Ei(1,-b/x^2)
```

maxima [A] time = 0.40, size = 24, normalized size = 0.96

$$\frac{1}{4}\,\mathrm{Ei}\left(-\frac{b}{x^2}\right)e^{(-a)}-\frac{1}{4}\,\mathrm{Ei}\left(\frac{b}{x^2}\right)e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*Ei(-b/x^2)*e^(-a) - 1/4*Ei(b/x^2)*e^a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$-\frac{\sinh(a)\,\mathrm{coshint}\left(\frac{b}{x^2}\right)}{2}-\frac{\cosh(a)\,\mathrm{sinhint}\left(\frac{b}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x,x)
```

```
[Out] - (sinh(a)*coshint(b/x^2))/2 - (cosh(a)*sinhint(b/x^2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x, x)
```

3.47 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{\pi}\,e^{-a}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,e^{a}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

```
[Out] 1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*erfi(b^(1/2)/x)*Pi^(1
/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5346, 5298, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^2,x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]/x])/(4*Sqrt[b]*E^a) - (E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/
(4*Sqrt[b])
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)-\frac{1}{2}\,\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)\\ &= \frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}-\frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.88

$$\frac{\sqrt{\pi}\left((\cosh(a)-\sinh(a))\text{erf}\left(\frac{\sqrt{b}}{x}\right)-(\sinh(a)+\cosh(a))\text{erfi}\left(\frac{\sqrt{b}}{x}\right)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^2,x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) - Erfi[Sqrt[b]/x]*(Cosh[a] +
Sinh[a])))/(4*Sqrt[b])
```

fricas [A] time = 0.54, size = 52, normalized size = 0.91

$$\frac{\sqrt{\pi}\,\sqrt{-b}\,(\cosh(a)+\sinh(a))\,\text{erf}\left(\frac{\sqrt{-b}}{x}\right)+\sqrt{\pi}\,\sqrt{b}\,(\cosh(a)-\sinh(a))\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)/x) + sqrt(pi)*sqrt(
b)*(cosh(a) - sinh(a))*erf(sqrt(b)/x))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^2, x)
```

maple [A] time = 0.04, size = 44, normalized size = 0.77

$$\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}\ \mathrm{e}^{-a}}{4\sqrt{b}}-\frac{\mathrm{e}^{a}\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{4\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^2,x)
```

```
[Out] 1/4*erf(b^(1/2)/x)*Pi^(1/2)*exp(-a)/b^(1/2)-1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*
erf((-b)^(1/2)/x)
```

maxima [A] time = 0.46, size = 62, normalized size = 1.09

$$-\frac{1}{2}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{3}{2},\frac{b}{x^2}\right)}{x^3\left(\frac{b}{x^2}\right)^{\frac{3}{2}}}+\frac{e^{a}\Gamma\left(\frac{3}{2},-\frac{b}{x^2}\right)}{x^3\left(-\frac{b}{x^2}\right)^{\frac{3}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(-a)*gamma(3/2, b/x^2)/(x^3*(b/x^2)^(3/2)) + e^a*gamma(3/2, -b/x^
2)/(x^3*(-b/x^2)^(3/2))) - sinh(a + b/x^2)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x^2,x)
```

```
[Out] int(sinh(a + b/x^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**2,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**2, x)
```

3.48 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3} dx$

Optimal. Leaf size=15

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -1/2*cosh(a+b/x^2)/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^3,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^3} dx &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \sinh(a+bx)\, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^3,x]
[Out] -1/2*Cosh[a + b/x^2]/b
```

fricas [A] time = 0.47, size = 17, normalized size = 1.13

$$-\frac{\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="fricas")
[Out] -1/2*cosh((a*x^2 + b)/x^2)/b
```

giac [B] time = 0.16, size = 27, normalized size = 1.80

$$-\frac{\left(e^{\left(2\,a+\frac{b}{x^2}\right)}+e^{\left(-\frac{b}{x^2}\right)}\right)e^{(-a)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="giac")
[Out] -1/4*(e^(2*a + b/x^2) + e^(-b/x^2))*e^(-a)/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^3,x)
[Out] -1/2*cosh(a+b/x^2)/b
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*cosh(a + b/x^2)/b
```

mupad [B] time = 0.37, size = 13, normalized size = 0.87

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x^3,x)
```

```
[Out] -cosh(a + b/x^2)/(2*b)
```

sympy [A] time = 2.74, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2b} & \text{for}\ b \neq 0 \\ -\frac{\sinh(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**3,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b), Ne(b, 0)), (-sinh(a)/(2*x**2), True))
```

3.49 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{\pi}\,e^{-a}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\,e^{a}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[Out] -1/2*cosh(a+b/x^2)/b/x+1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)+1/8*exp(a
)*erfi(b^(1/2)/x)*Pi^(1/2)/b^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5346, 5324, 5299, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^{-a}\operatorname{Erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{\sqrt{\pi}\,e^{a}\operatorname{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^4,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b*x) + (Sqrt[Pi]*Erf[Sqrt[b]/x])/(8*b^(3/2)*E^a) + (E^a
*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(8*b^(3/2))
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^4}\,dx &= -\text{Subst}\left(\int x^2\sinh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\text{Subst}\left(\int \cosh\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\ &= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx, x, \frac{1}{x}\right)}{4b}+\frac{\text{Subst}\left(\int e^{a+bx^2}\,dx, x, \frac{1}{x}\right)}{4b}\\ &= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}+\frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 74, normalized size = 0.99

$$\frac{\sqrt{\pi}\,x(\cosh(a)-\sinh(a))\text{erf}\left(\frac{\sqrt{b}}{x}\right)+\sqrt{\pi}\,x(\sinh(a)+\cosh(a))\text{erfi}\left(\frac{\sqrt{b}}{x}\right)-4\sqrt{b}\,\cosh\left(a+\frac{b}{x^2}\right)}{8b^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^4,x]
```

```
[Out] (-4*Sqrt[b]*Cosh[a + b/x^2] + Sqrt[Pi]*x*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a])
 + Sqrt[Pi]*x*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]))/(8*b^(3/2)*x)
```

fricas [B] time = 0.51, size = 251, normalized size = 3.35

$$-\frac{2\,b\cosh\left(\frac{ax^2+b}{x^2}\right)^2+\sqrt{\pi}\left(x\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right)+x\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a)+(x\cosh(a)+x\sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x^
2) + x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) + x*sinh(a))*sinh((a*x^2
+ b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) - sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/
x^2) - x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) - x*sinh(a))*sinh((a*x^
2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) + 4*b*cosh((a*x^2 + b)/x^2)*sinh((a*x^2
 + b)/x^2) + 2*b*sinh((a*x^2 + b)/x^2)^2 + 2*b)/(b^2*x*cosh((a*x^2 + b)/x^2
) + b^2*x*sinh((a*x^2 + b)/x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + \frac{b}{x^2}\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^4, x)
```

maple [A] time = 0.05, size = 82, normalized size = 1.09

$$-\frac{e^{-a} e^{-\frac{b}{x^2}}}{4 b x} + \frac{e^{-a} \sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8 b^{\frac{3}{2}}} - \frac{e^{a} e^{\frac{b}{x^2}}}{4 x b} + \frac{e^{a} \sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{8 b \sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^4,x)
```

```
[Out] -1/4*exp(-a)/b/x*exp(-b/x^2)+1/8*exp(-a)/b^(3/2)*Pi^(1/2)*erf(b^(1/2)/x)-1/
4*exp(a)*exp(b/x^2)/x/b+1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

maxima [A] time = 0.39, size = 62, normalized size = 0.83

$$-\frac{1}{6} b \left(\frac{e^{(-a)} \Gamma\left(\frac{5}{2}, \frac{b}{x^2}\right)}{x^5 \left(\frac{b}{x^2}\right)^{\frac{5}{2}}} + \frac{e^{a} \Gamma\left(\frac{5}{2}, -\frac{b}{x^2}\right)}{x^5 \left(-\frac{b}{x^2}\right)^{\frac{5}{2}}} \right) - \frac{\sinh\left(a + \frac{b}{x^2}\right)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*b*(e^(-a)*gamma(5/2, b/x^2)/(x^5*(b/x^2)^(5/2)) + e^a*gamma(5/2, -b/x^
2)/(x^5*(-b/x^2)^(5/2))) - 1/3*sinh(a + b/x^2)/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a + \frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x^4,x)
```

```
[Out] int(sinh(a + b/x^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + \frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**4,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**4, x)
```

3.50 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5} dx$

Optimal. Leaf size=34

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}$$

```
[Out] -1/2*cosh(a+b/x^2)/b/x^2+1/2*sinh(a+b/x^2)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 3296, 2637}

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^5,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b*x^2) + Sinh[a + b/x^2]/(2*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^5}\,dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int x\sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2} + \frac{\operatorname{Subst}\left(\int \cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{2b}$$
$$= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^2} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{2b^2}$$

Mathematica [**A**] time = 0.03, size = 34, normalized size = 1.00

$$\frac{x^2\sinh\left(a+\frac{b}{x^2}\right) - b\cosh\left(a+\frac{b}{x^2}\right)}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^5,x]
[Out] (-(b*Cosh[a + b/x^2]) + x^2*Sinh[a + b/x^2])/(2*b^2*x^2)
```

fricas [**A**] time = 0.54, size = 40, normalized size = 1.18

$$\frac{x^2\sinh\left(\frac{ax^2+b}{x^2}\right) - b\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="fricas")
[Out] 1/2*(x^2*sinh((a*x^2 + b)/x^2) - b*cosh((a*x^2 + b)/x^2))/(b^2*x^2)
```

giac [**A**] time = 0.19, size = 43, normalized size = 1.26

$$-\frac{\left(\left(\frac{b}{x^2}-1\right)e^{\left(2\,a+\frac{b}{x^2}\right)} + \left(\frac{b}{x^2}+1\right)e^{\left(-\frac{b}{x^2}\right)}\right)e^{(-a)}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="giac")
[Out] -1/4*((b/x^2 - 1)*e^(2*a + b/x^2) + (b/x^2 + 1)*e^(-b/x^2))*e^(-a)/b^2
```

maple [A] time = 0.03, size = 55, normalized size = 1.62

$$-\frac{\left(-x^{2}+b\right) \mathrm{e}^{\frac{a\,x^{2}+b}{x^{2}}}}{4x^{2}b^{2}}-\frac{\left(x^{2}+b\right) \mathrm{e}^{-\frac{a\,x^{2}+b}{x^{2}}}}{4x^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^5,x)
```

```
[Out] -1/4*(-x^2+b)/x^2/b^2*exp((a*x^2+b)/x^2)-1/4*(x^2+b)/x^2/b^2*exp(-(a*x^2+b)
/x^2)
```

maxima [C] time = 0.47, size = 48, normalized size = 1.41

$$-\frac{1}{8}\,b\left(\frac{e^{(-a)}\Gamma\left(3,\frac{b}{x^{2}}\right)}{b^{3}}-\frac{e^{a}\Gamma\left(3,-\frac{b}{x^{2}}\right)}{b^{3}}\right)-\frac{\sinh\left(a+\frac{b}{x^{2}}\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*(e^(-a)*gamma(3, b/x^2)/b^3 - e^a*gamma(3, -b/x^2)/b^3) - 1/4*sinh(a
 + b/x^2)/x^4
```

mupad [B] time = 0.41, size = 58, normalized size = 1.71

$$-\frac{\mathrm{e}^{a+\frac{b}{x^{2}}}\,\left(\frac{1}{4\,b}-\frac{x^{2}}{4\,b^{2}}\right)}{x^{2}}-\frac{\mathrm{e}^{-a-\frac{b}{x^{2}}}\,\left(\frac{1}{4\,b}+\frac{x^{2}}{4\,b^{2}}\right)}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x^5,x)
```

```
[Out] - (exp(a + b/x^2)*(1/(4*b) - x^2/(4*b^2)))/x^2 - (exp(- a - b/x^2)*(1/(4*b)
 + x^2/(4*b^2)))/x^2
```

sympy [A] time = 7.67, size = 37, normalized size = 1.09

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x^{2}}\right)}{2bx^{2}}+\frac{\sinh\left(a+\frac{b}{x^{2}}\right)}{2b^{2}} & \text{for}\: b \neq 0 \\ -\frac{\sinh\left(a\right)}{4x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**5,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b*x**2) + sinh(a + b/x**2)/(2*b**2), Ne(b,
0)), (-sinh(a)/(4*x**4), True))
```

3.51 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6} dx$

Optimal. Leaf size=93

$$\frac{3\sqrt{\pi}\, e^{-a}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}} - \frac{3\sqrt{\pi}\, e^{a}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}} + \frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}$$

```
[Out] -1/2*cosh(a+b/x^2)/b/x^3+3/4*sinh(a+b/x^2)/b^2/x+3/16*erf(b^(1/2)/x)*Pi^(1/
2)/b^(5/2)/exp(a)-3/16*exp(a)*erfi(b^(1/2)/x)*Pi^(1/2)/b^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5346, 5324, 5325, 5298, 2204, 2205}

$$\frac{3\sqrt{\pi}\, e^{-a}\text{Erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}} - \frac{3\sqrt{\pi}\, e^{a}\text{Erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}} + \frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^6,x]
```

```
[Out] -Cosh[a + b/x^2]/(2*b*x^3) + (3*Sqrt[Pi]*Erf[Sqrt[b]/x])/(16*b^(5/2)*E^a) -
 (3*E^a*Sqrt[Pi]*Erfi[Sqrt[b]/x])/(16*b^(5/2)) + (3*Sinh[a + b/x^2])/(4*b^2
*x)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5325

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Sinh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5346

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> -Subst[Int[(a + b*Sinh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx &= -\mathrm{Subst}\left(\int x^4\sinh\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3\,\mathrm{Subst}\left(\int x^2\cosh\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)}{2b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}-\frac{3\,\mathrm{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx,x,\frac{1}{x}\right)}{4b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}+\frac{3\,\mathrm{Subst}\left(\int e^{-a-bx^2}\,dx,x,\frac{1}{x}\right)}{8b^2}-\frac{3\,\mathrm{Subst}\left(\int e^{a+bx^2}\,dx,x,
\end{aligned}$$

$$= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^3}+\frac{3e^{-a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}-\frac{3e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)}{16b^{5/2}}+\frac{3\sinh\left(a+\frac{b}{x^2}\right)}{4b^2x}$$

Mathematica [A] time = 0.12, size = 97, normalized size = 1.04

$$\frac{3\sqrt{\pi}\,x^3(\cosh(a)-\sinh(a))\mathrm{erf}\left(\frac{\sqrt{b}}{x}\right)-3\sqrt{\pi}\,x^3(\sinh(a)+\cosh(a))\mathrm{erfi}\left(\frac{\sqrt{b}}{x}\right)+4\sqrt{b}\,\left(3x^2\sinh\left(a+\frac{b}{x^2}\right)-2b\cosh\left(\right.\right.}{16b^{5/2}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^6,x]
```

```
[Out] (3*Sqrt[Pi]*x^3*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) - 3*Sqrt[Pi]*x^3*Erfi[Sq
rt[b]/x]*(Cosh[a] + Sinh[a]) + 4*Sqrt[b]*(-2*b*Cosh[a + b/x^2] + 3*x^2*Sinh
[a + b/x^2]))/(16*b^(5/2)*x^3)
```

fricas [B] time = 0.45, size = 313, normalized size = 3.37

$$-\frac{6\,bx^2 - 2\left(3\,bx^2 - 2\,b^2\right)\cosh\left(\frac{ax^2+b}{x^2}\right)^2 - 3\,\sqrt{\pi}\left(x^3\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right) + x^3\cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a) + \left(x^3\cosh(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x^2 - 2*(3*b*x^2 - 2*b^2)*cosh((a*x^2 + b)/x^2)^2 - 3*sqrt(pi)*(
x^3*cosh(a)*cosh((a*x^2 + b)/x^2) + x^3*cosh((a*x^2 + b)/x^2)*sinh(a) + (x^
3*cosh(a) + x^3*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) -
3*sqrt(pi)*(x^3*cosh(a)*cosh((a*x^2 + b)/x^2) - x^3*cosh((a*x^2 + b)/x^2)*s
inh(a) + (x^3*cosh(a) - x^3*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqr
t(b)/x) - 4*(3*b*x^2 - 2*b^2)*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) -
 2*(3*b*x^2 - 2*b^2)*sinh((a*x^2 + b)/x^2)^2 + 4*b^2)/(b^3*x^3*cosh((a*x^2
+ b)/x^2) + b^3*x^3*sinh((a*x^2 + b)/x^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + \frac{b}{x^2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^6, x)
```

maple [A] time = 0.06, size = 117, normalized size = 1.26

$$-\frac{e^{-a}e^{-\frac{b}{x^2}}}{4b\,x^3} - \frac{3\,e^{-a}e^{-\frac{b}{x^2}}}{8b^2x} + \frac{3\,e^{-a}\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{16b^{\frac{5}{2}}} - \frac{e^{a}e^{\frac{b}{x^2}}}{4x^3b} + \frac{3\,e^{a}e^{\frac{b}{x^2}}}{8b^2x} - \frac{3\,e^{a}\sqrt{\pi}\ \operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{16b^2\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^6,x)
```

```
[Out] -1/4*exp(-a)/b/x^3*exp(-b/x^2)-3/8*exp(-a)/b^2/x*exp(-b/x^2)+3/16*exp(-a)/b
^(5/2)*Pi^(1/2)*erf(b^(1/2)/x)-1/4*exp(a)*exp(b/x^2)/x^3/b+3/8*exp(a)/b^2*e
xp(b/x^2)/x-3/16*exp(a)/b^2*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

maxima [A] time = 0.77, size = 62, normalized size = 0.67

$$-\frac{1}{10}\,b\left(\frac{e^{(-a)}\Gamma\left(\frac{7}{2},\frac{b}{x^2}\right)}{x^7\left(\frac{b}{x^2}\right)^{\frac{7}{2}}}+\frac{e^{a}\Gamma\left(\frac{7}{2},-\frac{b}{x^2}\right)}{x^7\left(-\frac{b}{x^2}\right)^{\frac{7}{2}}}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*b*(e^(-a)*gamma(7/2, b/x^2)/(x^7*(b/x^2)^(7/2)) + e^a*gamma(7/2, -b/x
^2)/(x^7*(-b/x^2)^(7/2))) - 1/5*sinh(a + b/x^2)/x^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x^6,x)
```

```
[Out] int(sinh(a + b/x^2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**6,x)
```

```
[Out] Integral(sinh(a + b/x**2)/x**6, x)
```

3.52 $\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7} \, dx$

Optimal. Leaf size=47

$$-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3}+\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}$$

```
[Out] -cosh(a+b/x^2)/b^3-1/2*cosh(a+b/x^2)/b/x^4+sinh(a+b/x^2)/b^2/x^2
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5320, 3296, 2638}

$$\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b/x^2]/x^7,x]
```

```
[Out] -(Cosh[a + b/x^2]/b^3) - Cosh[a + b/x^2]/(2*b*x^4) + Sinh[a + b/x^2]/(b^2*x
^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\operatorname{Subst}\left(\int x\cosh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{b}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2} - \frac{\operatorname{Subst}\left(\int \sinh(a+bx)\,dx, x, \frac{1}{x^2}\right)}{b^2}\\
&= -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} - \frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4} + \frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.94

$$\frac{2bx^2\sinh\left(a+\frac{b}{x^2}\right) - \left(b^2+2x^4\right)\cosh\left(a+\frac{b}{x^2}\right)}{2b^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b/x^2]/x^7,x]
[Out] (-((b^2 + 2*x^4)*Cosh[a + b/x^2]) + 2*b*x^2*Sinh[a + b/x^2])/(2*b^3*x^4)
```

fricas [A] time = 0.47, size = 50, normalized size = 1.06

$$\frac{2\,bx^2\sinh\left(\frac{ax^2+b}{x^2}\right) - \left(2\,x^4+b^2\right)\cosh\left(\frac{ax^2+b}{x^2}\right)}{2\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="fricas")
[Out] 1/2*(2*b*x^2*sinh((a*x^2 + b)/x^2) - (2*x^4 + b^2)*cosh((a*x^2 + b)/x^2))/(
b^3*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+\frac{b}{x^2}\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sinh(a + b/x^2)/x^7, x)
```

maple [A] time = 0.05, size = 73, normalized size = 1.55

$$-\frac{\left(2x^4-2b\,x^2+b^2\right)e^{\frac{a\,x^2+b}{x^2}}}{4b^3x^4}-\frac{\left(2x^4+2b\,x^2+b^2\right)e^{-\frac{a\,x^2+b}{x^2}}}{4b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b/x^2)/x^7,x)
```

```
[Out] -1/4*(2*x^4-2*b*x^2+b^2)/b^3/x^4*exp((a*x^2+b)/x^2)-1/4*(2*x^4+2*b*x^2+b^2)
/b^3/x^4*exp(-(a*x^2+b)/x^2)
```

maxima [C] time = 0.38, size = 47, normalized size = 1.00

$$-\frac{1}{12}\,b\left(\frac{e^{(-a)}\Gamma\left(4,\frac{b}{x^2}\right)}{b^4}+\frac{e^{a}\Gamma\left(4,-\frac{b}{x^2}\right)}{b^4}\right)-\frac{\sinh\left(a+\frac{b}{x^2}\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*b*(e^(-a)*gamma(4, b/x^2)/b^4 + e^a*gamma(4, -b/x^2)/b^4) - 1/6*sinh(
a + b/x^2)/x^6
```

mupad [B] time = 0.44, size = 74, normalized size = 1.57

$$-\frac{e^{a+\frac{b}{x^2}}\left(\frac{1}{4\,b}-\frac{x^2}{2\,b^2}+\frac{x^4}{2\,b^3}\right)}{x^4}-\frac{e^{-a-\frac{b}{x^2}}\left(\frac{1}{4\,b}+\frac{x^2}{2\,b^2}+\frac{x^4}{2\,b^3}\right)}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)/x^7,x)
```

```
[Out] - (exp(a + b/x^2)*(1/(4*b) - x^2/(2*b^2) + x^4/(2*b^3)))/x^4 - (exp(- a - b
/x^2)*(1/(4*b) + x^2/(2*b^2) + x^4/(2*b^3)))/x^4
```

sympy [A] time = 19.22, size = 51, normalized size = 1.09

$$\begin{cases} -\frac{\cosh\left(a+\frac{b}{x^2}\right)}{2bx^4}+\frac{\sinh\left(a+\frac{b}{x^2}\right)}{b^2x^2}-\frac{\cosh\left(a+\frac{b}{x^2}\right)}{b^3} & \text{for}\ b\neq 0\\ -\frac{\sinh\left(a\right)}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b/x**2)/x**7,x)
```

```
[Out] Piecewise((-cosh(a + b/x**2)/(2*b*x**4) + sinh(a + b/x**2)/(b**2*x**2) - co
sh(a + b/x**2)/b**3, Ne(b, 0)), (-sinh(a)/(6*x**6), True))
```

3.53 $\int (ex)^m \sinh^3\left(a + \frac{b}{x^2}\right) dx$

Optimal. Leaf size=194

$$\frac{1}{16}e^{3a}3^{\frac{m+1}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{3b}{x^2}\right)-\frac{3}{16}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)+\frac{3}{16}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(e$$

```
[Out] 1/16*3^(1/2+1/2*m)*exp(3*a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m
,-3*b/x^2)-3/16*exp(a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,-b/x
^2)+3/16*(b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,b/x^2)/exp(a)-1/16*
3^(1/2+1/2*m)*(b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,3*b/x^2)/exp(3
*a)
```

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5350, 5340, 5328, 2218}

$$\frac{1}{16}e^{3a}3^{\frac{m+1}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{3b}{x^2}\right)-\frac{3}{16}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)+\frac{3}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x^2]^3,x]
```

```
[Out] (3^((1 + m)/2)*E^(3*a)*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
-3*b)/x^2])/16 - (3*E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2,
-(b/x^2)])/16 + (3*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/
(16*E^a) - (3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, (
3*b)/x^2])/(16*E^(3*a))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int (ex)^m \sinh^3\left(a+\frac{b}{x^2}\right)dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh^3\left(a+bx^2\right)dx, x, \frac{1}{x}\right)\right)$$

$$= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int \left(-\frac{3}{4}x^{-2-m}\sinh\left(a+bx^2\right)+\frac{1}{4}x^{-2-m}\sinh\left(3a+3bx^2\right)\right) d$$

$$= -\left(\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh\left(3a+3bx^2\right)dx, x, \frac{1}{x}\right)\right)+\frac{1}{4}\left(3\left(\frac{1}{x}\right)^m (ex)^m$$

$$= \frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{-3a-3bx^2}x^{-2-m}dx, x, \frac{1}{x}\right)-\frac{1}{8}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{3a+}\right.$$

$$= \frac{1}{16}3^{\frac{1+m}{2}}e^{3a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{3b}{x^2}\right)-\frac{3}{16}e^{a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-\right.$$

Mathematica [B] time = 24.29, size = 1039, normalized size = 5.36

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2]^3,x]
```

```
[Out] ((e*x)^m*Cosh[a]^3*((-3*(((-(b/x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2
, -(b/x^2)])/2 - ((b/x^2)^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, b/x^2])/2
))/8 + ((3^((1 + m)/2)*(-(b/x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, (
-3*b)/x^2])/2 - (3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)
/2, (3*b)/x^2])/2)/8))/x^m + (3*x*(e*x)^m*Cosh[a]^2*(-4*Cosh[b/x^2] + 4*Cos
h[(3*b)/x^2] - 3^((1 + m)/2)*m*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, (-3
```

```
*b)/x^2] + m*(-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, -(b/x^2)] + m*(b/x^2)
^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2] - 3^((1 + m)/2)*m*(b/x^2)^((1 + m)/2)
*Gamma[(-1 - m)/2, (3*b)/x^2] - 2*3^((1 + m)/2)*(-(b/x^2))^((1 + m)/2)*Gamm
a[(1 - m)/2, (-3*b)/x^2] + 2*(-(b/x^2))^((1 + m)/2)*Gamma[(1 - m)/2, -(b/x^
2)] + 2*(b/x^2)^((1 + m)/2)*Gamma[(1 - m)/2, b/x^2] - 2*3^((1 + m)/2)*(b/x^
2)^((1 + m)/2)*Gamma[(1 - m)/2, (3*b)/x^2])*Sinh[a])/16 + ((e*x)^m*((3*(((-
(b/x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, -(b/x^2)])/2 + ((b/x^2)^((
1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, b/x^2])/2))/8 + ((3^((1 + m)/2)*(-(b/
x^2))^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, (-3*b)/x^2])/2 + (3^((1 + m)/
2)*(b/x^2)^((1 + m)/2)*x^(1 + m)*Gamma[(-1 - m)/2, (3*b)/x^2])/2)/8)*Sinh[a
]^3)/x^m + (3*x*(e*x)^m*Cosh[a]*Sinh[a]^2*(-(3^((1 + m)/2)*m*(-(b/x^2))^((1
 + m)/2)*Gamma[(-1 - m)/2, (-3*b)/x^2]) - m*(-(b/x^2))^((1 + m)/2)*Gamma[(-
1 - m)/2, -(b/x^2)] + m*(b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2] + 3^((
1 + m)/2)*m*(b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, (3*b)/x^2] - 2*3^((1 + m)
/2)*(-(b/x^2))^((1 + m)/2)*Gamma[(1 - m)/2, (-3*b)/x^2] - 2*(-(b/x^2))^((1
+ m)/2)*Gamma[(1 - m)/2, -(b/x^2)] + 2*(b/x^2)^((1 + m)/2)*Gamma[(1 - m)/2,
 b/x^2] + 2*3^((1 + m)/2)*(b/x^2)^((1 + m)/2)*Gamma[(1 - m)/2, (3*b)/x^2] +
 4*Sinh[b/x^2] + 4*Sinh[(3*b)/x^2]))/16
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax^2+b}{x^2}\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^3, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^3\left(a+\frac{b}{x^2}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x^2)^3,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x^2)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a + \frac{b}{x^2}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a + \frac{b}{x^2}\right)^3 (e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)^3*(e*x)^m,x)
```

```
[Out] int(sinh(a + b/x^2)^3*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^3\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x**2)**3,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x**2)**3, x)
```

3.54 $\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=117

$$e^{2a}2^{\frac{m-5}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{2b}{x^2}\right)+e^{-2a}2^{\frac{m-5}{2}}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),\frac{2b}{x^2}\right)-\frac{x(ex)^m}{2(m+1)}$$

```
[Out] -1/2*x*(e*x)^m/(1+m)+2^(-5/2+1/2*m)*exp(2*a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m
*GAMMA(-1/2-1/2*m,-2*b/x^2)+2^(-5/2+1/2*m)*(b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GA
MMA(-1/2-1/2*m,2*b/x^2)/exp(2*a)
```

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5350, 5340, 5329, 2218}

$$e^{2a}2^{\frac{m-5}{2}}x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{2b}{x^2}\right)+e^{-2a}2^{\frac{m-5}{2}}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),\frac{2b}{x^2}\right)-\frac{x(ex}{2(m}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x^2]^2,x]
```

```
[Out] -(x*(e*x)^m)/(2*(1 + m)) + 2^((-5 + m)/2)*E^(2*a)*(-(b/x^2))^((1 + m)/2)*x*
(e*x)^m*Gamma[(-1 - m)/2, (-2*b)/x^2] + (2^((-5 + m)/2)*(b/x^2)^((1 + m)/2)
*x*(e*x)^m*Gamma[(-1 - m)/2, (2*b)/x^2])/E^(2*a)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5329

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5340

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^2\left(a+\frac{b}{x^2}\right) dx &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\sinh^2\left(a+bx^2\right) dx, x, \frac{1}{x}\right)\right)\\ &= -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int \left(-\frac{1}{2}x^{-2-m}+\frac{1}{2}x^{-2-m}\cosh\left(2a+2bx^2\right)\right) dx, x, \frac{1}{x}\right)\right)\\ &= -\frac{x(ex)^m}{2(1+m)}-\frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\cosh\left(2a+2bx^2\right) dx, x, \frac{1}{x}\right)\\ &= -\frac{x(ex)^m}{2(1+m)}-\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int e^{-2a-2bx^2}x^{-2-m} dx, x, \frac{1}{x}\right)-\frac{1}{4}\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{S}\\ &= -\frac{x(ex)^m}{2(1+m)}+2^{\frac{1}{2}(-5+m)}e^{2a}\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}}x(ex)^m\Gamma\left(\frac{1}{2}(-1-m),-\frac{2b}{x^2}\right)+2^{\frac{1}{2}(-5+m)}e^{-2a}\left(\frac{b}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.83, size = 122, normalized size = 1.04

$$\frac{x(ex)^m\left(2^{\frac{m+1}{2}}(m+1)(\sinh(2a)+\cosh(2a))\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}\Gamma\left(\frac{1}{2}(-m-1),-\frac{2b}{x^2}\right)+2^{\frac{m+1}{2}}(m+1)(\cosh(2a)-\sinh(2a))\left(\frac{b}{x^2}\right.\right.}{8(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2]^2,x]
```

```
[Out] (x*(e*x)^m*(-4 + 2^((1 + m)/2)*(1 + m)*(b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2
, (2*b)/x^2]*(Cosh[2*a] - Sinh[2*a]) + 2^((1 + m)/2)*(1 + m)*(-(b/x^2))^((1
 + m)/2)*Gamma[(-1 - m)/2, (-2*b)/x^2]*(Cosh[2*a] + Sinh[2*a])))/(8*(1 + m)
)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m\sinh\left(\frac{ax^2+b}{x^2}\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a + \frac{b}{x^2}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2)^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^2\left(a + \frac{b}{x^2}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x^2)^2,x)
```

```
[Out] int((e*x)^m*sinh(a+b/x^2)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} e^m \int e^{\left(m \log(x) + 2a + \frac{2b}{x^2}\right)} dx + \frac{1}{4} e^m \int e^{\left(m \log(x) - 2a - \frac{2b}{x^2}\right)} dx - \frac{(ex)^{m+1}}{2e(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*e^m*integrate(e^(m*log(x) + 2*a + 2*b/x^2), x) + 1/4*e^m*integrate(e^(m
*log(x) - 2*a - 2*b/x^2), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a + \frac{b}{x^2}\right)^2 (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)^2*(e*x)^m,x)
[Out] int(sinh(a + b/x^2)^2*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^2\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x**2)**2,x)
[Out] Integral((e*x)**m*sinh(a + b/x**2)**2, x)
```

3.55 $\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=87

$$\frac{1}{4}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)-\frac{1}{4}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\Gamma\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right)$$

```
[Out] 1/4*exp(a)*(-b/x^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,-b/x^2)-1/4*(b/x
^2)^(1/2+1/2*m)*x*(e*x)^m*GAMMA(-1/2-1/2*m,b/x^2)/exp(a)
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5350, 5328, 2218}

$$\frac{1}{4}e^a x\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)-\frac{1}{4}e^{-a}x\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}(ex)^m\text{Gamma}\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b/x^2],x]
```

```
[Out] (E^a*(-(b/x^2))^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, -(b/x^2)])/4 - ((b/
x^2)^((1 + m)/2)*x*(e*x)^m*Gamma[(-1 - m)/2, b/x^2])/(4*E^a)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5328

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 5350

```
Int[((e_.)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[(a + b*Sinh[c + d/x^n])^p/
x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IntegerQ[p] &
& ILtQ[n, 0] &&  !RationalQ[m]
```

Rubi steps

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int x^{-2-m} \sinh\left(a+bx^2\right) dx, x, \frac{1}{x}\right)\right)$$
$$= \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{-a-bx^2} x^{-2-m} dx, x, \frac{1}{x}\right) - \frac{1}{2}\left(\left(\frac{1}{x}\right)^m (ex)^m\right) \text{Subst}\left(\int e^{a+bx^2} x$$
$$= \frac{1}{4}e^a\left(-\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m \Gamma\left(\frac{1}{2}(-1-m), -\frac{b}{x^2}\right) - \frac{1}{4}e^{-a}\left(\frac{b}{x^2}\right)^{\frac{1+m}{2}} x(ex)^m \Gamma\left(\frac{1}{2}(-1-m), \frac{b}{x^2}\right)$$

Mathematica [A] time = 0.14, size = 84, normalized size = 0.97

$$\frac{1}{4}x(ex)^m\left((\sinh(a)+\cosh(a))\left(-\frac{b}{x^2}\right)^{\frac{m+1}{2}}\Gamma\left(\frac{1}{2}(-m-1),-\frac{b}{x^2}\right)-(\cosh(a)-\sinh(a))\left(\frac{b}{x^2}\right)^{\frac{m+1}{2}}\Gamma\left(\frac{1}{2}(-m-1),\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b/x^2],x]
```

```
[Out] (x*(e*x)^m*(-((b/x^2)^((1 + m)/2)*Gamma[(-1 - m)/2, b/x^2]*(Cosh[a] - Sinh[
a])) + (-(b/x^2))^((1 + m)/2)*Gamma[(-1 - m)/2, -(b/x^2)]*(Cosh[a] + Sinh[a
])))/4
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \sinh\left(\frac{ax^2+b}{x^2}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh((a*x^2 + b)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2), x)
```

maple [C] time = 0.10, size = 77, normalized size = 0.89

$$\frac{(ex)^m\, b\, \text{hypergeom}\left(\left[\frac{1}{4}-\frac{m}{4}\right],\left[\frac{3}{2},\frac{5}{4}-\frac{m}{4}\right],\frac{b^2}{4x^4}\right)\cosh(a)}{(-1+m)\,x}+\frac{(ex)^m\, x\, \text{hypergeom}\left(\left[-\frac{1}{4}-\frac{m}{4}\right],\left[\frac{1}{2},\frac{3}{4}-\frac{m}{4}\right],\frac{b^2}{4x^4}\right)\sinh(}{1+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b/x^2),x)
```

```
[Out] (e*x)^m*b/(-1+m)/x*hypergeom([1/4-1/4*m],[3/2,5/4-1/4*m],1/4/x^4*b^2)*cosh(
a)+(e*x)^m/(1+m)*x*hypergeom([-1/4-1/4*m],[1/2,3/4-1/4*m],1/4/x^4*b^2)*sinh
(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(a + b/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+\frac{b}{x^2}\right) (e\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b/x^2)*(e*x)^m,x)
```

```
[Out] int(sinh(a + b/x^2)*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(a+\frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b/x**2),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b/x**2), x)
```

3.56 $\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=26

$$x^{-m}(ex)^m \mathrm{Int}\left(x^m \mathrm{csch}\left(a+\frac{b}{x^2}\right),x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(a+b/x^2),x)/(x^m)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b/x^2],x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b/x^2], x])/x^m
```

Rubi steps

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right)\,dx = \left(x^{-m}(ex)^m\right)\int x^m \mathrm{csch}\left(a+\frac{b}{x^2}\right)\,dx$$

Mathematica [A] time = 3.40, size = 0, normalized size = 0.00

$$\int (ex)^m \mathrm{csch}\left(a+\frac{b}{x^2}\right)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b/x^2],x]
```

```
[Out] Integrate[(e*x)^m*Csch[a + b/x^2], x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex)^m}{\sinh\left(\frac{ax^2+b}{x^2}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="fricas")
[Out] integral((e*x)^m/sinh((a*x^2 + b)/x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a + \frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="giac")
[Out] integrate((e*x)^m/sinh(a + b/x^2), x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a + \frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a+b/x^2),x)
[Out] int((e*x)^m/sinh(a+b/x^2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a + \frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b/x^2),x, algorithm="maxima")
[Out] integrate((e*x)^m/sinh(a + b/x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{\sinh\left(a + \frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a + b/x^2),x)
```

```
[Out] int((e*x)^m/sinh(a + b/x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh\left(a + \frac{b}{x^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(a+b/x**2),x)
```

```
[Out] Integral((e*x)**m/sinh(a + b/x**2), x)
```

3.57 $\int \frac{\sinh(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\cosh\left(\sqrt{x}\right)$$

```
[Out] 2*cosh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5320, 2638}

$$2\cosh\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Cosh[Sqrt[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \sinh(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cosh\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\cosh\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Cosh[Sqrt[x]]
```

fricas [A] time = 0.56, size = 6, normalized size = 0.75

$$2 \cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*cosh(sqrt(x))
```

giac [A] time = 0.17, size = 11, normalized size = 1.38

$$e^{\left(-\sqrt{x}\right)} + e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] e^(-sqrt(x)) + e^sqrt(x)
```

maple [A] time = 0.01, size = 7, normalized size = 0.88

$$2 \cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*cosh(x^(1/2))
```

maxima [A] time = 0.44, size = 6, normalized size = 0.75

$$2 \cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*cosh(sqrt(x))
```

mupad [B] time = 0.40, size = 6, normalized size = 0.75

$$2 \cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*cosh(x^(1/2))
```

sympy [A] time = 0.26, size = 7, normalized size = 0.88

$$2\cosh\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*cosh(sqrt(x))
```

3.58 $\int x^2 \sinh(a + bx^n)\, dx$

Optimal. Leaf size=75

$$\frac{e^{-a}x^3 (bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n}, bx^n\right)}{2n} - \frac{e^{a}x^3 (-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n}, -bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x^3*GAMMA(3/n,-b*x^n)/n/((-b*x^n)^(3/n))+1/2*x^3*GAMMA(3/n,b*x^
n)/exp(a)/n/((b*x^n)^(3/n))
```

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5360, 2218}

$$\frac{e^{-a}x^3 (bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n}, bx^n\right)}{2n} - \frac{e^{a}x^3 (-bx^n)^{-3/n}\,\text{Gamma}\left(\frac{3}{n}, -bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*x^3*Gamma[3/n, -(b*x^n)])/(2*n*(-(b*x^n))^(3/n)) + (x^3*Gamma[3/n, b*
x^n])/(2*E^a*n*(b*x^n)^(3/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int x^2 \sinh\left(a+bx^n\right)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}x^2\,dx\right)+\frac{1}{2}\int e^{a+bx^n}x^2\,dx$$
$$= -\frac{e^a x^3 \left(-bx^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-bx^n\right)}{2n}+\frac{e^{-a}x^3\left(bx^n\right)^{-3/n}\Gamma\left(\frac{3}{n},bx^n\right)}{2n}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 1.17

$$-\frac{x^3\left(-b^2x^{2n}\right)^{-3/n}\left((\sinh(a)+\cosh(a))\left(bx^n\right)^{3/n}\Gamma\left(\frac{3}{n},-bx^n\right)-(\cosh(a)-\sinh(a))\left(-bx^n\right)^{3/n}\Gamma\left(\frac{3}{n},bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(x^3*(-((-(b*x^n))^(3/n)*Gamma[3/n, b*x^n]*(Cosh[a] - Sinh[a])) + (b*x
^n)^(3/n)*Gamma[3/n, -(b*x^n)]*(Cosh[a] + Sinh[a])))/(n*(-(b^2*x^(2*n)))^(3
/n))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\sinh\left(bx^n+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(b*x^n + a), x)
```

maple [C] time = 0.13, size = 77, normalized size = 1.03

$$\frac{x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{2n}\right],\left[\frac{1}{2},1+\frac{3}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{3}+\frac{x^{n+3}b\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{3}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{3}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*x^n),x)
```

```
[Out] 1/3*x^3*hypergeom([3/2/n],[1/2,1+3/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+3)*x^
(n+3)*b*hypergeom([1/2+3/2/n],[3/2,3/2+3/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

maxima [A] time = 0.56, size = 73, normalized size = 0.97

$$\frac{x^3 e^{(-a)} \Gamma\left(\frac{3}{n}, bx^n\right)}{2 \left(bx^n\right)^{\frac{3}{n}} n} - \frac{x^3 e^a \Gamma\left(\frac{3}{n}, -bx^n\right)}{2 \left(-bx^n\right)^{\frac{3}{n}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^3*e^(-a)*gamma(3/n, b*x^n)/((b*x^n)^(3/n)*n) - 1/2*x^3*e^a*gamma(3/n,
 -b*x^n)/((-b*x^n)^(3/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sinh\left(a + b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*x^n),x)
```

```
[Out] int(x^2*sinh(a + b*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*x**n),x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n), x)
```

3.59 $\int x \sinh(a+bx^n)\,dx$

Optimal. Leaf size=75

$$\frac{e^{-a}x^2(bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},bx^n\right)}{2n}-\frac{e^{a}x^2(-bx^n)^{-2/n}\,\Gamma\left(\frac{2}{n},-bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x^2*GAMMA(2/n,-b*x^n)/n/((-b*x^n)^(2/n))+1/2*x^2*GAMMA(2/n,b*x^
n)/exp(a)/n/((b*x^n)^(2/n))
```

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5360, 2218}

$$\frac{e^{-a}x^2(bx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},bx^n\right)}{2n}-\frac{e^{a}x^2(-bx^n)^{-2/n}\,\text{Gamma}\left(\frac{2}{n},-bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*x^2*Gamma[2/n, -(b*x^n)])/(2*n*(-(b*x^n))^(2/n)) + (x^2*Gamma[2/n, b*
x^n])/(2*E^a*n*(b*x^n)^(2/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int x\sinh\left(a+bx^n\right)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}x\,dx\right)+\frac{1}{2}\int e^{a+bx^n}x\,dx$$
$$= -\frac{e^a x^2\left(-bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-bx^n\right)}{2n}+\frac{e^{-a}x^2\left(bx^n\right)^{-2/n}\Gamma\left(\frac{2}{n},bx^n\right)}{2n}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 1.17

$$-\frac{x^2\left(-b^2x^{2n}\right)^{-2/n}\left(\left(\sinh(a)+\cosh(a)\right)\left(bx^n\right)^{2/n}\Gamma\left(\frac{2}{n},-bx^n\right)-\left(\cosh(a)-\sinh(a)\right)\left(-bx^n\right)^{2/n}\Gamma\left(\frac{2}{n},bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^n],x]
[Out] -1/2*(x^2*(-((-(b*x^n))^(2/n)*Gamma[2/n, b*x^n]*(Cosh[a] - Sinh[a])) + (b*x
^n)^(2/n)*Gamma[2/n, -(b*x^n)]*(Cosh[a] + Sinh[a])))/(n*(-(b^2*x^(2*n)))^(2
/n))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(x\sinh\left(bx^n+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="fricas")
[Out] integral(x*sinh(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="giac")
[Out] integrate(x*sinh(b*x^n + a), x)
```

maple [C] time = 0.09, size = 69, normalized size = 0.92

$$\frac{x^2\,\text{hypergeom}\left(\left[\frac{1}{n}\right],\left[\frac{1}{2},1+\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{2}+\frac{x^{n+2}b\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*x^n),x)
```

```
[Out] 1/2*x^2*hypergeom([1/n],[1/2,1+1/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+2)*x^(n+2
)*b*hypergeom([1/2+1/n],[3/2,3/2+1/n],1/4*x^(2*n)*b^2)*cosh(a)
```

maxima [A] time = 0.51, size = 73, normalized size = 0.97

$$\frac{x^2e^{(-a)}\Gamma\left(\frac{2}{n}, bx^n\right)}{2\,(bx^n)^{\frac{2}{n}}\,n} - \frac{x^2e^a\Gamma\left(\frac{2}{n}, -bx^n\right)}{2\,(-bx^n)^{\frac{2}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*e^(-a)*gamma(2/n, b*x^n)/((b*x^n)^(2/n)*n) - 1/2*x^2*e^a*gamma(2/n,
 -b*x^n)/((-b*x^n)^(2/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sinh\left(a + b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^n),x)
```

```
[Out] int(x*sinh(a + b*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x**n),x)
```

```
[Out] Integral(x*sinh(a + b*x**n), x)
```

3.60 $\int \sinh(a+bx^n)\,dx$

Optimal. Leaf size=67

$$\frac{e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}-\frac{e^{a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}$$

```
[Out] -1/2*exp(a)*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))+1/2*x*GAMMA(1/n,b*x^n)/e
xp(a)/n/((b*x^n)^(1/n))
```

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5306, 2208}

$$\frac{e^{-a}x(bx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},bx^n\right)}{2n}-\frac{e^{a}x(-bx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},-bx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*x*Gamma[n^(-1), -(b*x^n)])/(2*n*(-(b*x^n))^n^(-1)) + (x*Gamma[n^(-1),
 b*x^n])/(2*E^a*n*(b*x^n)^n^(-1))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5306

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sinh(a+bx^n)\,dx &= -\left(\frac{1}{2}\int e^{-a-bx^n}\,dx\right)+\frac{1}{2}\int e^{a+bx^n}\,dx\\ &= -\frac{e^{a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{2n}+\frac{e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{2n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 79, normalized size = 1.18

$$\frac{\left(-b^2x^{2n}\right)^{-1/n}\left(x(\cosh(a)-\sinh(a))\left(-bx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},bx^n\right)-x(\sinh(a)+\cosh(a))\left(bx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n],x]
```

```
[Out] (x*(-(b*x^n))^n^(-1)*Gamma[n^(-1), b*x^n]*(Cosh[a] - Sinh[a]) - x*(b*x^n)^n
^(-1)*Gamma[n^(-1), -(b*x^n)]*(Cosh[a] + Sinh[a]))/(2*n*(-(b^2*x^(2*n)))^n^
(-1))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sinh\left(bx^n+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a), x)
```

maple [C] time = 0.07, size = 74, normalized size = 1.10

$$x\,\mathrm{hypergeom}\left(\left[\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)+\frac{x^{n+1}b\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n),x)
```

```
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(n+1)*x^(n+1)*
b*hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

maxima [A] time = 0.52, size = 61, normalized size = 0.91

$$\frac{xe^{(-a)}\Gamma\left(\frac{1}{n}, bx^n\right)}{2\,(bx^n)^{\left(\frac{1}{n}\right)}\,n} - \frac{xe^{a}\Gamma\left(\frac{1}{n}, -bx^n\right)}{2\,(-bx^n)^{\left(\frac{1}{n}\right)}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 1/2*x*e^a*gamma(1/n, -b*
x^n)/((-b*x^n)^(1/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a + b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n),x)
```

```
[Out] int(sinh(a + b*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n),x)
```

```
[Out] Integral(sinh(a + b*x**n), x)
```

3.61 $\int \frac{\sinh(a+bx^n)}{x} \, dx$

Optimal. Leaf size=25

$$\frac{\sinh(a)\text{Chi}\left(bx^n\right)}{n}+\frac{\cosh(a)\text{Shi}\left(bx^n\right)}{n}$$

```
[Out] cosh(a)*Shi(b*x^n)/n+Chi(b*x^n)*sinh(a)/n
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5318, 5317, 5316}

$$\frac{\sinh(a)\text{Chi}\left(bx^n\right)}{n}+\frac{\cosh(a)\text{Shi}\left(bx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]/x,x]
```

```
[Out] (CoshIntegral[b*x^n]*Sinh[a])/n + (Cosh[a]*SinhIntegral[b*x^n])/n
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh\left(a+bx^n\right)}{x}\,dx &= \cosh(a)\int \frac{\sinh\left(bx^n\right)}{x}\,dx+\sinh(a)\int \frac{\cosh\left(bx^n\right)}{x}\,dx\\ &= \frac{\text{Chi}\left(bx^n\right)\sinh(a)}{n}+\frac{\cosh(a)\text{Shi}\left(bx^n\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.92

$$\frac{\sinh(a)\text{Chi}\left(bx^n\right)+\cosh(a)\text{Shi}\left(bx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]/x,x]
```

```
[Out] (CoshIntegral[b*x^n]*Sinh[a] + Cosh[a]*SinhIntegral[b*x^n])/n
```

fricas [B] time = 0.58, size = 55, normalized size = 2.20

$$\frac{(\cosh(a)+\sinh(a))\text{Ei}\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)\right)-(\cosh(a)-\sinh(a))\text{Ei}\left(-b\cosh\left(n\log(x)\right)-b\sin\right.}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*((cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) - (cosh(a)
 - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sinh\left(bx^n+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)/x, x)
```

maple [A] time = 0.02, size = 33, normalized size = 1.32

$$\frac{\mathrm{e}^{-a}\,\text{Ei}\left(1,b\,x^n\right)}{2n}-\frac{\mathrm{e}^{a}\,\text{Ei}\left(1,-b\,x^n\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)/x,x)
```

```
[Out] 1/2/n*exp(-a)*Ei(1,b*x^n)-1/2/n*exp(a)*Ei(1,-b*x^n)
```

maxima [A] time = 0.61, size = 30, normalized size = 1.20

$$-\frac{\text{Ei}\left(-bx^n\right)e^{(-a)}}{2\,n}+\frac{\text{Ei}\left(bx^n\right)e^{a}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*Ei(-b*x^n)*e^(-a)/n + 1/2*Ei(b*x^n)*e^a/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sinh(a + b\,x^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)/x,x)
```

```
[Out] int(sinh(a + b*x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(a + bx^n)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)/x,x)
```

```
[Out] Integral(sinh(a + b*x**n)/x, x)
```

3.62 $\int \frac{\sinh(a+bx^n)}{x^2}\,dx$

Optimal. Leaf size=71

$$\frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx}-\frac{e^{a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

```
[Out] -1/2*exp(a)*(-b*x^n)^(1/n)*GAMMA(-1/n,-b*x^n)/n/x+1/2*(b*x^n)^(1/n)*GAMMA(-
1/n,b*x^n)/exp(a)/n/x
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5360, 2218}

$$\frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},bx^n\right)}{2nx}-\frac{e^{a}(-bx^n)^{\frac{1}{n}}\,\text{Gamma}\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]/x^2,x]
```

```
[Out] -(E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(2*n*x) + ((b*x^n)^n^(-1)
*Gamma[-n^(-1), b*x^n])/(2*E^a*n*x)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh(a+bx^n)}{x^2}\,dx = -\left(\frac{1}{2}\int \frac{e^{-a-bx^n}}{x^2}\,dx\right) + \frac{1}{2}\int \frac{e^{a+bx^n}}{x^2}\,dx$$
$$= -\frac{e^a(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx} + \frac{e^{-a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)}{2nx}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.96

$$\frac{(\cosh(a)-\sinh(a))(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},bx^n\right)-(\sinh(a)+\cosh(a))(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-bx^n\right)}{2nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]/x^2,x]
```

```
[Out] ((b*x^n)^n^(-1)*Gamma[-n^(-1), b*x^n]*(Cosh[a] - Sinh[a]) - (-(b*x^n))^n^(-
1)*Gamma[-n^(-1), -(b*x^n)]*(Cosh[a] + Sinh[a]))/(2*n*x)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx^n+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx^n+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)/x^2, x)
```

maple [C] time = 0.11, size = 77, normalized size = 1.08

$$-\frac{\text{hypergeom}\left(\left[-\frac{1}{2n}\right],\left[\frac{1}{2},1-\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{x}+\frac{x^{-1+n}b\,\text{hypergeom}\left(\left[\frac{1}{2}-\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}-\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{-1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)/x^2,x)
```

```
[Out] -1/x*hypergeom([-1/2/n],[1/2,1-1/2/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(-1+n)*x^(
-1+n)*b*hypergeom([1/2-1/2/n],[3/2,3/2-1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

maxima [A] time = 0.69, size = 65, normalized size = 0.92

$$\frac{(bx^n)^{\left(\frac{1}{n}\right)}e^{(-a)}\Gamma\left(-\frac{1}{n}, bx^n\right)}{2\,nx} - \frac{(-bx^n)^{\left(\frac{1}{n}\right)}e^{a}\Gamma\left(-\frac{1}{n}, -bx^n\right)}{2\,nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^n)^(1/n)*e^(-a)*gamma(-1/n, b*x^n)/(n*x) - 1/2*(-b*x^n)^(1/n)*e^a*
gamma(-1/n, -b*x^n)/(n*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a + b\,x^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)/x^2,x)
```

```
[Out] int(sinh(a + b*x^n)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + bx^{n}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)/x**2, x)
```

3.63 $\int \frac{\sinh(a+bx^n)}{x^3} dx$

Optimal. Leaf size=75

$$\frac{e^{-a}(bx^n)^{2/n}\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2}-\frac{e^{a}(-bx^n)^{2/n}\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

```
[Out] -1/2*exp(a)*(-b*x^n)^(2/n)*GAMMA(-2/n,-b*x^n)/n/x^2+1/2*(b*x^n)^(2/n)*GAMMA
(-2/n,b*x^n)/exp(a)/n/x^2
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5360, 2218}

$$\frac{e^{-a}(bx^n)^{2/n}\text{Gamma}\left(-\frac{2}{n},bx^n\right)}{2nx^2}-\frac{e^{a}(-bx^n)^{2/n}\text{Gamma}\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]/x^3,x]
```

```
[Out] -(E^a*(-(b*x^n))^(2/n)*Gamma[-2/n, -(b*x^n)])/(2*n*x^2) + ((b*x^n)^(2/n)*Ga
mma[-2/n, b*x^n])/(2*E^a*n*x^2)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int \frac{\sinh(a+bx^n)}{x^3}\,dx = -\left(\frac{1}{2}\int \frac{e^{-a-bx^n}}{x^3}\,dx\right) + \frac{1}{2}\int \frac{e^{a+bx^n}}{x^3}\,dx$$
$$= -\frac{e^a(-bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2} + \frac{e^{-a}(bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},bx^n\right)}{2nx^2}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.96

$$\frac{(\cosh(a)-\sinh(a))\,(bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},bx^n\right) - (\sinh(a)+\cosh(a))\,(-bx^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-bx^n\right)}{2nx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]/x^3,x]
[Out] ((b*x^n)^(2/n)*Gamma[-2/n, b*x^n]*(Cosh[a] - Sinh[a]) - (-(b*x^n))^(2/n)*Ga
mma[-2/n, -(b*x^n)]*(Cosh[a] + Sinh[a]))/(2*n*x^2)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx^n+a)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="fricas")
[Out] integral(sinh(b*x^n + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx^n+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)/x^3, x)
```

maple [C] time = 0.05, size = 77, normalized size = 1.03

$$-\frac{\text{hypergeom}\left(\left[-\frac{1}{n}\right],\left[\frac{1}{2},1-\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{2x^2} + \frac{x^{-2+n}b\,\text{hypergeom}\left(\left[\frac{1}{2}-\frac{1}{n}\right],\left[\frac{3}{2},\frac{3}{2}-\frac{1}{n}\right],\frac{x^{2n}b^2}{4}\right)\cosh(a)}{-2+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)/x^3,x)
```

```
[Out] -1/2/x^2*hypergeom([-1/n],[1/2,1-1/n],1/4*x^(2*n)*b^2)*sinh(a)+1/(-2+n)*x^(
-2+n)*b*hypergeom([1/2-1/n],[3/2,3/2-1/n],1/4*x^(2*n)*b^2)*cosh(a)
```

maxima [A] time = 0.67, size = 69, normalized size = 0.92

$$\frac{\left(bx^{n}\right)^{\frac{2}{n}} e^{(-a)}\Gamma\left(-\frac{2}{n}, bx^{n}\right)}{2\, nx^{2}} - \frac{\left(-bx^{n}\right)^{\frac{2}{n}} e^{a}\Gamma\left(-\frac{2}{n}, -bx^{n}\right)}{2\, nx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^n)^(2/n)*e^(-a)*gamma(-2/n, b*x^n)/(n*x^2) - 1/2*(-b*x^n)^(2/n)*e^
a*gamma(-2/n, -b*x^n)/(n*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+b\,x^n\right)}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)/x^3,x)
```

```
[Out] int(sinh(a + b*x^n)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh{\left(a + bx^{n} \right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)/x**3,x)
```

```
[Out] Integral(sinh(a + b*x**n)/x**3, x)
```

3.64 $\int x^2 \sinh^2(a + bx^n) \, dx$

Optimal. Leaf size=99

$$-\frac{e^{2a}2^{-\frac{3}{n}-2}x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{3}{n}-2}x^3(bx^n)^{-3/n}\Gamma\left(\frac{3}{n},2bx^n\right)}{n}-\frac{x^3}{6}$$

```
[Out] -1/6*x^3-2^(-2-3/n)*exp(2*a)*x^3*GAMMA(3/n,-2*b*x^n)/n/((-b*x^n)^(3/n))-2^(
-2-3/n)*x^3*GAMMA(3/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(3/n))
```

Rubi [A] time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}2^{-\frac{3}{n}-2}x^3(-bx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{3}{n}-2}x^3(bx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},2bx^n\right)}{n}-\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^n]^2,x]
```

```
[Out] -x^3/6 - (2^(-2 - 3/n)*E^(2*a)*x^3*Gamma[3/n, -2*b*x^n])/(n*(-(b*x^n))^(3/n
)) - (2^(-2 - 3/n)*x^3*Gamma[3/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(3/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\sinh^2(a+bx^n)\,dx=\int\left(-\frac{x^2}{2}+\frac{1}{2}x^2\cosh(2a+2bx^n)\right)dx$$
$$=-\frac{x^3}{6}+\frac{1}{2}\int x^2\cosh(2a+2bx^n)\,dx$$
$$=-\frac{x^3}{6}+\frac{1}{4}\int e^{-2a-2bx^n}x^2\,dx+\frac{1}{4}\int e^{2a+2bx^n}x^2\,dx$$
$$=-\frac{x^3}{6}-\frac{2^{-2-\frac{3}{n}}e^{2a}x^3(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{3}{n}}e^{-2a}x^3(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2bx^n\right)}{n}$$

Mathematica [A] time = 1.51, size = 89, normalized size = 0.90

$$-\frac{x^3\left(3e^{2a}8^{-1/n}(-bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},-2bx^n\right)+3e^{-2a}8^{-1/n}(bx^n)^{-3/n}\,\Gamma\left(\frac{3}{n},2bx^n\right)+2n\right)}{12n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^n]^2,x]
[Out] -1/12*(x^3*(2*n + (3*E^(2*a)*Gamma[3/n, -2*b*x^n])/(8^n^(-1)*(-(b*x^n))^(3/
n)) + (3*Gamma[3/n, 2*b*x^n])/(8^n^(-1)*E^(2*a)*(b*x^n)^(3/n))))/n
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^2\sinh(bx^n+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral(x^2*sinh(b*x^n + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh(bx^n+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="giac")
[Out] integrate(x^2*sinh(b*x^n + a)^2, x)
```

maple [F]   time = 0.13, size = 0, normalized size = 0.00

$$\int x^2 \left(\sinh^2(a + b\,x^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*x^n)^2,x)
```

```
[Out] int(x^2*sinh(a+b*x^n)^2,x)
```

maxima [A]   time = 0.44, size = 82, normalized size = 0.83

$$-\frac{1}{6}x^3 - \frac{x^3 e^{(-2\,a)}\Gamma\left(\frac{3}{n}, 2\,bx^n\right)}{4\,(2\,bx^n)^{\frac{3}{n}}\,n} - \frac{x^3 e^{(2\,a)}\Gamma\left(\frac{3}{n}, -2\,bx^n\right)}{4\,(-2\,bx^n)^{\frac{3}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/6*x^3 - 1/4*x^3*e^(-2*a)*gamma(3/n, 2*b*x^n)/((2*b*x^n)^(3/n)*n) - 1/4*x
^3*e^(2*a)*gamma(3/n, -2*b*x^n)/((-2*b*x^n)^(3/n)*n)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sinh(a + b\,x^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*x^n)^2,x)
```

```
[Out] int(x^2*sinh(a + b*x^n)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh^2(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n)**2, x)
```

3.65 $\int x \sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=99

$$-\frac{e^{2a}4^{-\frac{1}{n}-1}x^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{e^{-2a}4^{-\frac{1}{n}-1}x^2(bx^n)^{-2/n}\Gamma\left(\frac{2}{n},2bx^n\right)}{n}-\frac{x^2}{4}$$

```
[Out] -1/4*x^2-4^(-1-1/n)*exp(2*a)*x^2*GAMMA(2/n,-2*b*x^n)/n/((-b*x^n)^(2/n))-4^(
-1-1/n)*x^2*GAMMA(2/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(2/n))
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}4^{-\frac{1}{n}-1}x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{e^{-2a}4^{-\frac{1}{n}-1}x^2(bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},2bx^n\right)}{n}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^n]^2,x]
```

```
[Out] -x^2/4 - (4^(-1 - n^(-1))*E^(2*a)*x^2*Gamma[2/n, -2*b*x^n])/(n*(-(b*x^n))^(
2/n)) - (4^(-1 - n^(-1))*x^2*Gamma[2/n, 2*b*x^n])/(E^(2*a)*n*(b*x^n)^(2/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x\sinh^2(a+bx^n)\,dx = \int \left(-\frac{x}{2}+\frac{1}{2}x\cosh(2a+2bx^n)\right)dx$$
$$= -\frac{x^2}{4}+\frac{1}{2}\int x\cosh(2a+2bx^n)\,dx$$
$$= -\frac{x^2}{4}+\frac{1}{4}\int e^{-2a-2bx^n}x\,dx+\frac{1}{4}\int e^{2a+2bx^n}x\,dx$$
$$= -\frac{x^2}{4}-\frac{4^{-1-\frac{1}{n}}e^{2a}x^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-2bx^n\right)}{n}-\frac{4^{-1-\frac{1}{n}}e^{-2a}x^2(bx^n)^{-2/n}\Gamma\left(\frac{2}{n},2bx^n\right)}{n}$$

Mathematica [A] time = 1.31, size = 85, normalized size = 0.86

$$-\frac{x^2\left(e^{2a}4^{-1/n}(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-2bx^n\right)+e^{-2a}4^{-1/n}(bx^n)^{-2/n}\Gamma\left(\frac{2}{n},2bx^n\right)+n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x^2*(n + (E^(2*a)*Gamma[2/n, -2*b*x^n])/(4^n^(-1)*(-(b*x^n))^(2/n)) +
 Gamma[2/n, 2*b*x^n]/(4^n^(-1)*E^(2*a)*(b*x^n)^(2/n))))/n
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x\sinh(bx^n+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x^n + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh(bx^n+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sinh(b*x^n + a)^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x\left(\sinh^2\left(a+b\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*x^n)^2,x)
```

```
[Out] int(x*sinh(a+b*x^n)^2,x)
```

maxima [A] time = 0.42, size = 82, normalized size = 0.83

$$-\frac{1}{4}\,x^2-\frac{x^2e^{(-2\,a)}\Gamma\left(\frac{2}{n},2\,bx^n\right)}{4\,\left(2\,bx^n\right)^{\frac{2}{n}}\,n}-\frac{x^2e^{(2\,a)}\Gamma\left(\frac{2}{n},-2\,bx^n\right)}{4\,\left(-2\,bx^n\right)^{\frac{2}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x^2 - 1/4*x^2*e^(-2*a)*gamma(2/n, 2*b*x^n)/((2*b*x^n)^(2/n)*n) - 1/4*x
^2*e^(2*a)*gamma(2/n, -2*b*x^n)/((-2*b*x^n)^(2/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sinh\left(a+b\,x^n\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^n)^2,x)
```

```
[Out] int(x*sinh(a + b*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^2\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(x*sinh(a + b*x**n)**2, x)
```

3.66 $\int \sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=89

$$-\frac{e^{2a}2^{-\frac{1}{n}-2}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{1}{n}-2}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)}{n}-\frac{x}{2}$$

```
[Out] -1/2*x-2^(-2-1/n)*exp(2*a)*x*GAMMA(1/n,-2*b*x^n)/n/((-b*x^n)^(1/n))-2^(-2-1
/n)*x*GAMMA(1/n,2*b*x^n)/exp(2*a)/n/((b*x^n)^(1/n))
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5308, 5307, 2208}

$$-\frac{e^{2a}2^{-\frac{1}{n}-2}x(-bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{e^{-2a}2^{-\frac{1}{n}-2}x(bx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},2bx^n\right)}{n}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^2,x]
```

```
[Out] -x/2 - (2^(-2 - n^(-1))*E^(2*a)*x*Gamma[n^(-1), -2*b*x^n])/(n*(-(b*x^n))^n^
(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), 2*b*x^n])/(E^(2*a)*n*(b*x^n)^n^(-1
))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5307

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5308

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rubi steps

$$\int \sinh^2(a+bx^n)\,dx = \int \left(-\frac{1}{2}+\frac{1}{2}\cosh(2a+2bx^n)\right)dx$$
$$= -\frac{x}{2}+\frac{1}{2}\int \cosh(2a+2bx^n)\,dx$$
$$= -\frac{x}{2}+\frac{1}{4}\int e^{-2a-2bx^n}\,dx+\frac{1}{4}\int e^{2a+2bx^n}\,dx$$
$$= -\frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2a}x(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2a}x(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)}{n}$$

Mathematica [A] time = 1.11, size = 81, normalized size = 0.91

$$-\frac{x\left(e^{2a}2^{-1/n}(-bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2bx^n\right)+e^{-2a}2^{-1/n}(bx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2bx^n\right)+2n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x*(2*n + (E^(2*a)*Gamma[n^(-1), -2*b*x^n])/(2^n^(-1)*(-(b*x^n))^n^(-1
)) + Gamma[n^(-1), 2*b*x^n]/(2^n^(-1)*E^(2*a)*(b*x^n)^n^(-1))))/n
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\sinh(bx^n+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh(bx^n+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \sinh^2(a + b\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^2,x)
```

```
[Out] int(sinh(a+b*x^n)^2,x)
```

maxima [A] time = 0.42, size = 68, normalized size = 0.76

$$-\frac{1}{2}\,x - \frac{xe^{(-2\,a)}\Gamma\left(\frac{1}{n}, 2\,bx^n\right)}{4\,\left(2\,bx^n\right)^{\left(\frac{1}{n}\right)}\,n} - \frac{xe^{(2\,a)}\Gamma\left(\frac{1}{n}, -2\,bx^n\right)}{4\,\left(-2\,bx^n\right)^{\left(\frac{1}{n}\right)}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/4*x*e^(-2*a)*gamma(1/n, 2*b*x^n)/((2*b*x^n)^(1/n)*n) - 1/4*x*e^(
2*a)*gamma(1/n, -2*b*x^n)/((-2*b*x^n)^(1/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a + b\,x^n\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^2,x)
```

```
[Out] int(sinh(a + b*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^2\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**2, x)
```

3.67 $\int \frac{\sinh^2(a+bx^n)}{x} \, dx$

Optimal. Leaf size=43

$$\frac{\cosh(2a)\text{Chi}\left(2bx^n\right)}{2n}+\frac{\sinh(2a)\text{Shi}\left(2bx^n\right)}{2n}-\frac{\log(x)}{2}$$

```
[Out] 1/2*Chi(2*b*x^n)*cosh(2*a)/n-1/2*ln(x)+1/2*Shi(2*b*x^n)*sinh(2*a)/n
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5362, 5319, 5317, 5316}

$$\frac{\cosh(2a)\text{Chi}\left(2bx^n\right)}{2n}+\frac{\sinh(2a)\text{Shi}\left(2bx^n\right)}{2n}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^2/x,x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n])/(2*n) - Log[x]/2 + (Sinh[2*a]*SinhIntegra
l[2*b*x^n])/(2*n)
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5319

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sinh^2(a+bx^n)}{x}\,dx = \int \left(-\frac{1}{2x}+\frac{\cosh(2a+2bx^n)}{2x}\right)dx$$
$$= -\frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cosh(2a+2bx^n)}{x}\,dx$$
$$= -\frac{\log(x)}{2}+\frac{1}{2}\cosh(2a)\int \frac{\cosh(2bx^n)}{x}\,dx+\frac{1}{2}\sinh(2a)\int \frac{\sinh(2bx^n)}{x}\,dx$$
$$= \frac{\cosh(2a)\text{Chi}(2bx^n)}{2n}-\frac{\log(x)}{2}+\frac{\sinh(2a)\text{Shi}(2bx^n)}{2n}$$

Mathematica [**A**] time = 0.04, size = 39, normalized size = 0.91

$$\frac{\cosh(2a)\text{Chi}(2bx^n)+\sinh(2a)\text{Shi}(2bx^n)}{2n}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^2/x,x]
```

```
[Out] -1/2*Log[x] + (Cosh[2*a]*CoshIntegral[2*b*x^n] + Sinh[2*a]*SinhIntegral[2*b
*x^n])/(2*n)
```

fricas [**A**] time = 0.54, size = 69, normalized size = 1.60

$$\frac{(\cosh(2a)+\sinh(2a))\text{Ei}\left(2b\cosh\left(n\log(x)\right)+2b\sinh\left(n\log(x)\right)\right)+(\cosh(2a)-\sinh(2a))\text{Ei}\left(-2b\cosh\left(n\log\right.\right.}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*((cosh(2*a) + sinh(2*a))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) +
(cosh(2*a) - sinh(2*a))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - 2*n*
log(x))/n
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx^n+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^2/x, x)
```

maple [A] time = 0.14, size = 40, normalized size = 0.93

$$-\frac{\ln(x)}{2}-\frac{e^{-2a}\,\mathrm{Ei}\left(1,2b\,x^{n}\right)}{4n}-\frac{e^{2a}\,\mathrm{Ei}\left(1,-2b\,x^{n}\right)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^2/x,x)
[Out] -1/2*ln(x)-1/4/n*exp(-2*a)*Ei(1,2*b*x^n)-1/4/n*exp(2*a)*Ei(1,-2*b*x^n)
```

maxima [A] time = 0.40, size = 37, normalized size = 0.86

$$\frac{\mathrm{Ei}\left(2\,bx^{n}\right)e^{(2\,a)}}{4\,n}+\frac{\mathrm{Ei}\left(-2\,bx^{n}\right)e^{(-2\,a)}}{4\,n}-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x,x, algorithm="maxima")
[Out] 1/4*Ei(2*b*x^n)*e^(2*a)/n + 1/4*Ei(-2*b*x^n)*e^(-2*a)/n - 1/2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(a+b\,x^{n}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^2/x,x)
[Out] int(sinh(a + b*x^n)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^{2}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**2/x,x)
[Out] Integral(sinh(a + b*x**n)**2/x, x)
```

3.68 $\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx$

Optimal. Leaf size=91

$$-\frac{e^{2a}2^{\frac{1}{n}-2}\left(-bx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{e^{-2a}2^{\frac{1}{n}-2}\left(bx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},2bx^n\right)}{nx}+\frac{1}{2x}$$

```
[Out] 1/2/x-2^(-2+1/n)*exp(2*a)*(-b*x^n)^(1/n)*GAMMA(-1/n,-2*b*x^n)/n/x-2^(-2+1/n
)*(b*x^n)^(1/n)*GAMMA(-1/n,2*b*x^n)/exp(2*a)/n/x
```

Rubi [A] time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}2^{\frac{1}{n}-2}\left(-bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{e^{-2a}2^{\frac{1}{n}-2}\left(bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},2bx^n\right)}{nx}+\frac{1}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^2/x^2,x]
```

```
[Out] 1/(2*x) - (2^(-2 + n^(-1))*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^
n])/(n*x) - (2^(-2 + n^(-1))*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/(E^(2*
a)*n*x)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx &= \int \left(-\frac{1}{2x^2}+\frac{\cosh(2a+2bx^n)}{2x^2}\right)dx\\
&= \frac{1}{2x}+\frac{1}{2}\int \frac{\cosh(2a+2bx^n)}{x^2}\,dx\\
&= \frac{1}{2x}+\frac{1}{4}\int \frac{e^{-2a-2bx^n}}{x^2}\,dx+\frac{1}{4}\int \frac{e^{2a+2bx^n}}{x^2}\,dx\\
&= \frac{1}{2x}-\frac{2^{-2+\frac{1}{n}}e^{2a}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2bx^n\right)}{nx}-\frac{2^{-2+\frac{1}{n}}e^{-2a}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2bx^n\right)}{nx}
\end{aligned}$$

Mathematica [A] time = 1.43, size = 79, normalized size = 0.87

$$-\frac{e^{2a}2^{\frac{1}{n}}(-bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},-2bx^n\right)+e^{-2a}2^{\frac{1}{n}}(bx^n)^{\frac{1}{n}}\,\Gamma\left(-\frac{1}{n},2bx^n\right)-2n}{4nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^2/x^2,x]
```

```
[Out] -1/4*(-2*n + 2^n^(-1)*E^(2*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -2*b*x^n] +
(2^n^(-1)*(b*x^n)^n^(-1)*Gamma[-n^(-1), 2*b*x^n])/E^(2*a))/(n*x)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx^n+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^2/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx^n+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^2/x^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+b\,x^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^2/x^2,x)
```

```
[Out] int(sinh(a+b*x^n)^2/x^2,x)
```

maxima [A] time = 0.43, size = 74, normalized size = 0.81

$$-\frac{(2\,bx^n)^{\left(\frac{1}{n}\right)}e^{(-2\,a)}\Gamma\left(-\frac{1}{n},2\,bx^n\right)}{4\,nx}-\frac{(-2\,bx^n)^{\left(\frac{1}{n}\right)}e^{(2\,a)}\Gamma\left(-\frac{1}{n},-2\,bx^n\right)}{4\,nx}+\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^n)^(1/n)*e^(-2*a)*gamma(-1/n, 2*b*x^n)/(n*x) - 1/4*(-2*b*x^n)^(
1/n)*e^(2*a)*gamma(-1/n, -2*b*x^n)/(n*x) + 1/2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a+b\,x^n)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^2/x^2,x)
```

```
[Out] int(sinh(a + b*x^n)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(a+bx^n)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**2/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**2/x**2, x)
```

3.69 $\int x^2 \sinh^3(a+bx^n)\, dx$

Optimal. Leaf size=166

$$-\frac{e^{3a}3^{-3/n}x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-3bx^n\right)}{8n}+\frac{3e^a x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^3(bx^n)^{-3/n}\Gamma\left(\frac{3}{n},bx^n\right)}{8n}+\frac{e^{-3a}3^{-3/n}x^3(bx^n)^{-3/n}\Gamma\left(\frac{3}{n},3bx^n\right)}{8n}$$

```
[Out] -1/8*exp(3*a)*x^3*GAMMA(3/n,-3*b*x^n)/(3^(3/n))/n/((-b*x^n)^(3/n))+3/8*exp(
a)*x^3*GAMMA(3/n,-b*x^n)/n/((-b*x^n)^(3/n))-3/8*x^3*GAMMA(3/n,b*x^n)/exp(a)
/n/((b*x^n)^(3/n))+1/8*x^3*GAMMA(3/n,3*b*x^n)/(3^(3/n))/exp(3*a)/n/((b*x^n)
^(3/n))
```

Rubi [A] time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}3^{-3/n}x^3(-bx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},-3bx^n\right)}{8n}+\frac{3e^a x^3(-bx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^3(bx^n)^{-3/n}\text{Gamma}\left(\frac{3}{n},bx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*x^3*Gamma[3/n, -3*b*x^n])/(8*3^(3/n)*n*(-(b*x^n))^(3/n)) + (3*E^a
*x^3*Gamma[3/n, -(b*x^n)])/(8*n*(-(b*x^n))^(3/n)) - (3*x^3*Gamma[3/n, b*x^n
])/(8*E^a*n*(b*x^n)^(3/n)) + (x^3*Gamma[3/n, 3*b*x^n])/(8*3^(3/n)*E^(3*a)*n
*(b*x^n)^(3/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \sinh^3(a+bx^n)\,dx &= \int \left(-\frac{3}{4}x^2\sinh(a+bx^n)+\frac{1}{4}x^2\sinh(3a+3bx^n)\right)dx\\ &= \frac{1}{4}\int x^2\sinh(3a+3bx^n)\,dx-\frac{3}{4}\int x^2\sinh(a+bx^n)\,dx\\ &= -\left(\frac{1}{8}\int e^{-3a-3bx^n}x^2\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}x^2\,dx+\frac{3}{8}\int e^{-a-bx^n}x^2\,dx-\frac{3}{8}\int e^{a+bx^n}x^2\,dx\\ &= -\frac{3^{-3/n}e^{3a}x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-3bx^n\right)}{8n}+\frac{3e^a x^3(-bx^n)^{-3/n}\Gamma\left(\frac{3}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^3(bx^n)^{-3/n}}{8n}\end{aligned}$$

Mathematica [A] time = 1.54, size = 161, normalized size = 0.97

$$-\frac{e^{-3a}27^{-1/n}x^3\left(-b^2x^{2n}\right)^{-3/n}\left((-bx^n)^{3/n}\left(e^{2a}3^{\frac{n+3}{n}}\Gamma\left(\frac{3}{n},bx^n\right)-\Gamma\left(\frac{3}{n},3bx^n\right)\right)+e^{6a}(bx^n)^{3/n}\Gamma\left(\frac{3}{n},-3bx^n\right)-e^{4a}3^{\frac{n+3}{n}}(bx^n\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x^3*(E^(6*a)*(b*x^n)^(3/n)*Gamma[3/n, -3*b*x^n] - 3^((3 + n)/n)*E^(4*
a)*(b*x^n)^(3/n)*Gamma[3/n, -(b*x^n)] + (-(b*x^n))^(3/n)*(3^((3 + n)/n)*E^(
2*a)*Gamma[3/n, b*x^n] - Gamma[3/n, 3*b*x^n])))/(27^n^(-1)*E^(3*a)*n*(-(b^2
*x^(2*n)))^(3/n))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2\sinh(bx^n+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*sinh(b*x^n + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sinh(bx^n+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sinh(b*x^n + a)^3, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x^2 \left(\sinh^3 (a + b\,x^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*x^n)^3,x)
```

```
[Out] int(x^2*sinh(a+b*x^n)^3,x)
```

maxima [A] time = 0.50, size = 149, normalized size = 0.90

$$\frac{x^3 e^{(-3\,a)}\Gamma\left(\frac{3}{n}, 3\,bx^n\right)}{8\,(3\,bx^n)^{\frac{3}{n}}\,n} - \frac{3\,x^3 e^{(-a)}\Gamma\left(\frac{3}{n}, bx^n\right)}{8\,(bx^n)^{\frac{3}{n}}\,n} + \frac{3\,x^3 e^a\Gamma\left(\frac{3}{n}, -bx^n\right)}{8\,(-bx^n)^{\frac{3}{n}}\,n} - \frac{x^3 e^{(3\,a)}\Gamma\left(\frac{3}{n}, -3\,bx^n\right)}{8\,(-3\,bx^n)^{\frac{3}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 1/8*x^3*e^(-3*a)*gamma(3/n, 3*b*x^n)/((3*b*x^n)^(3/n)*n) - 3/8*x^3*e^(-a)*g
amma(3/n, b*x^n)/((b*x^n)^(3/n)*n) + 3/8*x^3*e^a*gamma(3/n, -b*x^n)/((-b*x^
n)^(3/n)*n) - 1/8*x^3*e^(3*a)*gamma(3/n, -3*b*x^n)/((-3*b*x^n)^(3/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sinh\left(a + b\,x^n\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*x^n)^3,x)
```

```
[Out] int(x^2*sinh(a + b*x^n)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh^3\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*x**n)**3,x)
```

```
[Out] Integral(x**2*sinh(a + b*x**n)**3, x)
```

3.70 $\int x \sinh^3(a + bx^n)\, dx$

Optimal. Leaf size=166

$$-\frac{e^{3a}9^{-1/n}x^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^a x^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^2(bx^n)^{-2/n}\Gamma\left(\frac{2}{n},bx^n\right)}{8n}+\frac{e^{-3a}9^{-1/n}x^2(bx^n)}{8n}$$

```
[Out] -1/8*exp(3*a)*x^2*GAMMA(2/n,-3*b*x^n)/(9^(1/n))/n/((-b*x^n)^(2/n))+3/8*exp(
a)*x^2*GAMMA(2/n,-b*x^n)/n/((-b*x^n)^(2/n))-3/8*x^2*GAMMA(2/n,b*x^n)/exp(a)
/n/((b*x^n)^(2/n))+1/8*x^2*GAMMA(2/n,3*b*x^n)/(9^(1/n))/exp(3*a)/n/((b*x^n)
^(2/n))
```

Rubi [A] time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}9^{-1/n}x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^a x^2(-bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^2(bx^n)^{-2/n}\text{Gamma}\left(\frac{2}{n},b\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*x^2*Gamma[2/n, -3*b*x^n])/(8*9^n^(-1)*n*(-(b*x^n))^(2/n)) + (3*E^
a*x^2*Gamma[2/n, -(b*x^n)])/(8*n*(-(b*x^n))^(2/n)) - (3*x^2*Gamma[2/n, b*x^
n])/(8*E^a*n*(b*x^n)^(2/n)) + (x^2*Gamma[2/n, 3*b*x^n])/(8*9^n^(-1)*E^(3*a)
*n*(b*x^n)^(2/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x\sinh^3(a+bx^n)\,dx = \int\left(-\frac{3}{4}x\sinh(a+bx^n)+\frac{1}{4}x\sinh(3a+3bx^n)\right)dx$$
$$= \frac{1}{4}\int x\sinh(3a+3bx^n)\,dx - \frac{3}{4}\int x\sinh(a+bx^n)\,dx$$
$$= -\left(\frac{1}{8}\int e^{-3a-3bx^n}x\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}x\,dx+\frac{3}{8}\int e^{-a-bx^n}x\,dx-\frac{3}{8}\int e^{a+bx^n}x\,dx$$
$$= -\frac{9^{-1/n}e^{3a}x^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-3bx^n\right)}{8n}+\frac{3e^ax^2(-bx^n)^{-2/n}\Gamma\left(\frac{2}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x^2(bx^n)^{-2/}}{8n}$$

Mathematica [A] time = 1.60, size = 161, normalized size = 0.97

$$-\frac{e^{-3a}9^{-1/n}x^2\left(-b^2x^{2n}\right)^{-2/n}\left((-bx^n)^{2/n}\left(e^{2a}3^{\frac{n+2}{n}}\Gamma\left(\frac{2}{n},bx^n\right)-\Gamma\left(\frac{2}{n},3bx^n\right)\right)+e^{6a}(bx^n)^{2/n}\Gamma\left(\frac{2}{n},-3bx^n\right)-e^{4a}3^{\frac{n+2}{n}}(bx^}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x^2*(E^(6*a)*(b*x^n)^(2/n)*Gamma[2/n, -3*b*x^n] - 3^((2 + n)/n)*E^(4*
a)*(b*x^n)^(2/n)*Gamma[2/n, -(b*x^n)] + (-(b*x^n))^(2/n)*(3^((2 + n)/n)*E^(
2*a)*Gamma[2/n, b*x^n] - Gamma[2/n, 3*b*x^n])))/(9^n^(-1)*E^(3*a)*n*(-(b^2*
x^(2*n)))^(2/n))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(x\sinh(bx^n+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x*sinh(b*x^n + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh(bx^n+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x*sinh(b*x^n + a)^3, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x\left(\sinh^3\left(a+b\,x^n\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*x^n)^3,x)
[Out] int(x*sinh(a+b*x^n)^3,x)
```

maxima [A] time = 0.50, size = 149, normalized size = 0.90

$$\frac{x^2e^{(-3\,a)}\Gamma\left(\frac{2}{n},3\,bx^n\right)}{8\,\left(3\,bx^n\right)^{\frac{2}{n}}\,n}-\frac{3\,x^2e^{(-a)}\Gamma\left(\frac{2}{n},bx^n\right)}{8\,\left(bx^n\right)^{\frac{2}{n}}\,n}+\frac{3\,x^2e^{a}\Gamma\left(\frac{2}{n},-bx^n\right)}{8\,\left(-bx^n\right)^{\frac{2}{n}}\,n}-\frac{x^2e^{(3\,a)}\Gamma\left(\frac{2}{n},-3\,bx^n\right)}{8\,\left(-3\,bx^n\right)^{\frac{2}{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x^n)^3,x, algorithm="maxima")
[Out] 1/8*x^2*e^(-3*a)*gamma(2/n, 3*b*x^n)/((3*b*x^n)^(2/n)*n) - 3/8*x^2*e^(-a)*g
amma(2/n, b*x^n)/((b*x^n)^(2/n)*n) + 3/8*x^2*e^a*gamma(2/n, -b*x^n)/((-b*x^
n)^(2/n)*n) - 1/8*x^2*e^(3*a)*gamma(2/n, -3*b*x^n)/((-3*b*x^n)^(2/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sinh\left(a+b\,x^n\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*x^n)^3,x)
[Out] int(x*sinh(a + b*x^n)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh^3\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*x**n)**3,x)
[Out] Integral(x*sinh(a + b*x**n)**3, x)
```

3.71 $\int \sinh^3(a+bx^n)\,dx$

Optimal. Leaf size=150

$$-\frac{e^{3a}3^{-1/n}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^{a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},bx^n\right)}{8n}+\frac{e^{-3a}3^{-1/n}x(bx^n)^{-1/}}{8n}$$

```
[Out] -1/8*exp(3*a)*x*GAMMA(1/n,-3*b*x^n)/(3^(1/n))/n/((-b*x^n)^(1/n))+3/8*exp(a)
*x*GAMMA(1/n,-b*x^n)/n/((-b*x^n)^(1/n))-3/8*x*GAMMA(1/n,b*x^n)/exp(a)/n/((b
*x^n)^(1/n))+1/8*x*GAMMA(1/n,3*b*x^n)/(3^(1/n))/exp(3*a)/n/((b*x^n)^(1/n))
```

Rubi [A] time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5308, 5306, 2208}

$$-\frac{e^{3a}3^{-1/n}x(-bx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^{a}x(-bx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},bx\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*x*Gamma[n^(-1), -3*b*x^n])/(8*3^n^(-1)*n*(-(b*x^n))^n^(-1)) + (3*
E^a*x*Gamma[n^(-1), -(b*x^n)])/(8*n*(-(b*x^n))^n^(-1)) - (3*x*Gamma[n^(-1),
 b*x^n])/(8*E^a*n*(b*x^n)^n^(-1)) + (x*Gamma[n^(-1), 3*b*x^n])/(8*3^n^(-1)*
E^(3*a)*n*(b*x^n)^n^(-1))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 5306

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5308

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_), x_Symbol] :> Int[Ex
pandTrigReduce[(a + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sinh^3(a+bx^n)\,dx &= \int \left(-\frac{3}{4}\sinh(a+bx^n)+\frac{1}{4}\sinh(3a+3bx^n)\right)dx \\
&= \frac{1}{4}\int \sinh(3a+3bx^n)\,dx - \frac{3}{4}\int \sinh(a+bx^n)\,dx \\
&= -\left(\frac{1}{8}\int e^{-3a-3bx^n}\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}\,dx+\frac{3}{8}\int e^{-a-bx^n}\,dx-\frac{3}{8}\int e^{a+bx^n}\,dx \\
&= -\frac{3^{-1/n}e^{3a}x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3bx^n\right)}{8n}+\frac{3e^a x(-bx^n)^{-1/n}\Gamma\left(\frac{1}{n},-bx^n\right)}{8n}-\frac{3e^{-a}x(bx^n)^{-1/n}\Gamma\left(\frac{1}{n},\right.}{8n}
\end{aligned}
$$

Mathematica [A] time = 1.21, size = 140, normalized size = 0.93

$$
\frac{e^{-3a}3^{-1/n}x\left(-b^2x^{2n}\right)^{-1/n}\left((-bx^n)^{\frac{1}{n}}\left(\Gamma\left(\frac{1}{n},3bx^n\right)-e^{2a}3^{\frac{1}{n}+1}\Gamma\left(\frac{1}{n},bx^n\right)\right)-e^{6a}(bx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-3bx^n\right)+e^{4a}3^{\frac{1}{n}+1}(bx^n)^{\frac{1}{n}}\Gamma\left(\right.}{8n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^3,x]
```

```
[Out] (x*(-(E^(6*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -3*b*x^n]) + 3^(1 + n^(-1))*E^(4
*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -(b*x^n)] + (-(b*x^n))^n^(-1)*(-(3^(1 + n^
(-1))*E^(2*a)*Gamma[n^(-1), b*x^n]) + Gamma[n^(-1), 3*b*x^n])))/(8*3^n^(-1)
*E^(3*a)*n*(-(b^2*x^(2*n)))^n^(-1))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$
\text{integral}\left(\sinh\left(bx^n+a\right)^3, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sinh\left(bx^n+a\right)^3\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(sinh(b*x^n + a)^3, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \sinh^{3}\left(a+b\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^3,x)
[Out] int(sinh(a+b*x^n)^3,x)
```

maxima [A] time = 0.50, size = 125, normalized size = 0.83

$$\frac{xe^{(-3\,a)}\Gamma\left(\frac{1}{n},3\,bx^{n}\right)}{8\,\left(3\,bx^{n}\right)^{\left(\frac{1}{n}\right)}n}-\frac{3\,xe^{(-a)}\Gamma\left(\frac{1}{n},bx^{n}\right)}{8\,\left(bx^{n}\right)^{\left(\frac{1}{n}\right)}n}+\frac{3\,xe^{a}\Gamma\left(\frac{1}{n},-bx^{n}\right)}{8\,\left(-bx^{n}\right)^{\left(\frac{1}{n}\right)}n}-\frac{xe^{(3\,a)}\Gamma\left(\frac{1}{n},-3\,bx^{n}\right)}{8\,\left(-3\,bx^{n}\right)^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3,x, algorithm="maxima")
[Out] 1/8*x*e^(-3*a)*gamma(1/n, 3*b*x^n)/((3*b*x^n)^(1/n)*n) - 3/8*x*e^(-a)*gamma
(1/n, b*x^n)/((b*x^n)^(1/n)*n) + 3/8*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^(1/
n)*n) - 1/8*x*e^(3*a)*gamma(1/n, -3*b*x^n)/((-3*b*x^n)^(1/n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{sinh}\left(a+b\,x^n\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^3,x)
[Out] int(sinh(a + b*x^n)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh^{3}{\left(a+bx^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**3,x)
[Out] Integral(sinh(a + b*x**n)**3, x)
```

3.72 $\int \frac{\sinh^3(a+bx^n)}{x}\,dx$

Optimal. Leaf size=67

$$-\frac{3\sinh(a)\text{Chi}\,(bx^n)}{4n}+\frac{\sinh(3a)\text{Chi}\,(3bx^n)}{4n}-\frac{3\cosh(a)\text{Shi}\,(bx^n)}{4n}+\frac{\cosh(3a)\text{Shi}\,(3bx^n)}{4n}$$

```
[Out] -3/4*cosh(a)*Shi(b*x^n)/n+1/4*cosh(3*a)*Shi(3*b*x^n)/n-3/4*Chi(b*x^n)*sinh(
a)/n+1/4*Chi(3*b*x^n)*sinh(3*a)/n
```

Rubi [A] time = 0.10, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5362, 5318, 5317, 5316}

$$-\frac{3\sinh(a)\text{Chi}\,(bx^n)}{4n}+\frac{\sinh(3a)\text{Chi}\,(3bx^n)}{4n}-\frac{3\cosh(a)\text{Shi}\,(bx^n)}{4n}+\frac{\cosh(3a)\text{Shi}\,(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a])/(4*n) + (CoshIntegral[3*b*x^n]*Sinh[3*a])/
(4*n) - (3*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (Cosh[3*a]*SinhIntegral[3*b
*x^n])/(4*n)
```

Rule 5316

```
Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinhIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5317

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CoshIntegral[d*x^n]/n, x]
 /; FreeQ[{d, n}, x]
```

Rule 5318

```
Int[Sinh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Sinh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Cosh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(a+bx^n)}{x}\,dx &= \int \left(-\frac{3\sinh(a+bx^n)}{4x}+\frac{\sinh(3a+3bx^n)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx^n)}{x}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^n)}{x}\,dx\\ &= -\left(\frac{1}{4}(3\cosh(a))\int \frac{\sinh(bx^n)}{x}\,dx\right)+\frac{1}{4}\cosh(3a)\int \frac{\sinh(3bx^n)}{x}\,dx-\frac{1}{4}(3\sinh(a))\int \frac{\mathrm{co}}{}\\ &= -\frac{3\mathrm{Chi}(bx^n)\sinh(a)}{4n}+\frac{\mathrm{Chi}(3bx^n)\sinh(3a)}{4n}-\frac{3\cosh(a)\mathrm{Shi}(bx^n)}{4n}+\frac{\cosh(3a)\mathrm{Shi}(3bx}{4n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.78

$$\frac{-3\sinh(a)\mathrm{Chi}(bx^n)+\sinh(3a)\mathrm{Chi}(3bx^n)-3\cosh(a)\mathrm{Shi}(bx^n)+\cosh(3a)\mathrm{Shi}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^3/x,x]
```

```
[Out] (-3*CoshIntegral[b*x^n]*Sinh[a] + CoshIntegral[3*b*x^n]*Sinh[3*a] - 3*Cosh[
a]*SinhIntegral[b*x^n] + Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n)
```

fricas [A] time = 0.60, size = 115, normalized size = 1.72

$$\frac{(\cosh(3a)+\sinh(3a))\mathrm{Ei}\left(3b\cosh\left(n\log(x)\right)+3b\sinh\left(n\log(x)\right)\right)-3(\cosh(a)+\sinh(a))\mathrm{Ei}\left(b\cosh\left(n\log(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/8*((cosh(3*a) + sinh(3*a))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) -
3*(cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*(cosh(a)
- sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) - (cosh(3*a) - sinh(3*a
))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx^n+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^3/x, x)
```

maple [A] time = 0.19, size = 67, normalized size = 1.00

$$\frac{e^{-3a}\operatorname{Ei}(1,3b\,x^n)}{8n}-\frac{3\,e^{-a}\operatorname{Ei}(1,b\,x^n)}{8n}-\frac{e^{3a}\operatorname{Ei}(1,-3b\,x^n)}{8n}+\frac{3\,e^{a}\operatorname{Ei}(1,-b\,x^n)}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^3/x,x)
```

```
[Out] 1/8/n*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a)*Ei(1,b*x^n)-1/8/n*exp(3*a)*Ei(1
,-3*b*x^n)+3/8/n*exp(a)*Ei(1,-b*x^n)
```

maxima [A] time = 0.43, size = 62, normalized size = 0.93

$$\frac{\operatorname{Ei}(3\,bx^n)\,e^{(3\,a)}}{8\,n}+\frac{3\operatorname{Ei}(-bx^n)\,e^{(-a)}}{8\,n}-\frac{\operatorname{Ei}(-3\,bx^n)\,e^{(-3\,a)}}{8\,n}-\frac{3\operatorname{Ei}(bx^n)\,e^{a}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(3*b*x^n)*e^(3*a)/n + 3/8*Ei(-b*x^n)*e^(-a)/n - 1/8*Ei(-3*b*x^n)*e^(-
3*a)/n - 3/8*Ei(b*x^n)*e^a/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+b\,x^n\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^3/x,x)
```

```
[Out] int(sinh(a + b*x^n)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3\left(a+bx^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**3/x,x)
```

```
[Out] Integral(sinh(a + b*x**n)**3/x, x)
```

3.73 $\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx$

Optimal. Leaf size=154

$$-\frac{e^{3a}3^{\frac{1}{n}}\left(-bx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^a\left(-bx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8nx}-\frac{3e^{-a}\left(bx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},bx^n\right)}{8nx}+\frac{e^{-3a}3^{\frac{1}{n}}\left(bx^n\right)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},3bx^n\right)}{8nx}$$

```
[Out] -1/8*3^(1/n)*exp(3*a)*(-b*x^n)^(1/n)*GAMMA(-1/n,-3*b*x^n)/n/x+3/8*exp(a)*(-
b*x^n)^(1/n)*GAMMA(-1/n,-b*x^n)/n/x-3/8*(b*x^n)^(1/n)*GAMMA(-1/n,b*x^n)/exp
(a)/n/x+1/8*3^(1/n)*(b*x^n)^(1/n)*GAMMA(-1/n,3*b*x^n)/exp(3*a)/n/x
```

Rubi [A] time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}3^{\frac{1}{n}}\left(-bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^a\left(-bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},-bx^n\right)}{8nx}-\frac{3e^{-a}\left(bx^n\right)^{\frac{1}{n}}\text{Gamma}\left(-\frac{1}{n},bx^n\right)}{8nx}+\frac{e^{-3a}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x^n]^3/x^2,x]
```

```
[Out] -(3^n^(-1)*E^(3*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n])/(8*n*x) + (3
*E^a*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)])/(8*n*x) - (3*(b*x^n)^n^(-1
)*Gamma[-n^(-1), b*x^n])/(8*E^a*n*x) + (3^n^(-1)*(b*x^n)^n^(-1)*Gamma[-n^(-
1), 3*b*x^n])/(8*E^(3*a)*n*x)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(a+bx^n)}{x^2}\,dx &= \int \left(-\frac{3\sinh(a+bx^n)}{4x^2}+\frac{\sinh(3a+3bx^n)}{4x^2}\right)dx\\ &= \frac{1}{4}\int \frac{\sinh(3a+3bx^n)}{x^2}\,dx-\frac{3}{4}\int \frac{\sinh(a+bx^n)}{x^2}\,dx\\ &= -\left(\frac{1}{8}\int \frac{e^{-3a-3bx^n}}{x^2}\,dx\right)+\frac{1}{8}\int \frac{e^{3a+3bx^n}}{x^2}\,dx+\frac{3}{8}\int \frac{e^{-a-bx^n}}{x^2}\,dx-\frac{3}{8}\int \frac{e^{a+bx^n}}{x^2}\,dx\\ &= -\frac{3^{\frac{1}{n}}e^{3a}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-3bx^n\right)}{8nx}+\frac{3e^a(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\right)}{8nx}-\frac{3e^{-a}(bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},bx^n\right)}{8nx}+\end{aligned}$$

Mathematica [A] time = 1.41, size = 126, normalized size = 0.82

$$\frac{e^{-3a}\left(e^{6a}\left(-3^{\frac{1}{n}}\right)(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-3bx^n\right)+3e^{4a}(-bx^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\right)+(bx^n)^{\frac{1}{n}}\left(3^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},3bx^n\right)-3e^{2a}\Gamma\left(-\frac{1}{n},bx^n\right)\right)\right)}{8nx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x^n]^3/x^2,x]
```

```
[Out] (-(3^n^(-1)*E^(6*a)*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -3*b*x^n]) + 3*E^(4*a)
*(-(b*x^n))^n^(-1)*Gamma[-n^(-1), -(b*x^n)] + (b*x^n)^n^(-1)*(-3*E^(2*a)*Ga
mma[-n^(-1), b*x^n] + 3^n^(-1)*Gamma[-n^(-1), 3*b*x^n]))/(8*E^(3*a)*n*x)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh(bx^n+a)^3}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(sinh(b*x^n + a)^3/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(bx^n+a)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x^n + a)^3/x^2, x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a + b x^n)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*x^n)^3/x^2,x)
```

```
[Out] int(sinh(a+b*x^n)^3/x^2,x)
```

maxima [A] time = 0.46, size = 133, normalized size = 0.86

$$\frac{(3\,bx^n)^{\left(\frac{1}{n}\right)} e^{(-3\,a)} \Gamma\left(-\frac{1}{n}, 3\,bx^n\right)}{8\,nx} - \frac{3\,(bx^n)^{\left(\frac{1}{n}\right)} e^{(-a)} \Gamma\left(-\frac{1}{n}, bx^n\right)}{8\,nx} + \frac{3\,(-bx^n)^{\left(\frac{1}{n}\right)} e^{a} \Gamma\left(-\frac{1}{n}, -bx^n\right)}{8\,nx} - \frac{(-3\,bx^n)^{\left(\frac{1}{n}\right)} e^{(3\,a)} \Gamma\left(-\frac{1}{n},\right.}{8\,nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x^n)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*x^n)^(1/n)*e^(-3*a)*gamma(-1/n, 3*b*x^n)/(n*x) - 3/8*(b*x^n)^(1/n)
*e^(-a)*gamma(-1/n, b*x^n)/(n*x) + 3/8*(-b*x^n)^(1/n)*e^a*gamma(-1/n, -b*x^
n)/(n*x) - 1/8*(-3*b*x^n)^(1/n)*e^(3*a)*gamma(-1/n, -3*b*x^n)/(n*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(a + b x^n)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^3/x^2,x)
```

```
[Out] int(sinh(a + b*x^n)^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(a + bx^n)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*x**n)**3/x**2,x)
```

```
[Out] Integral(sinh(a + b*x**n)**3/x**2, x)
```

3.74 $\int (ex)^m (b\sinh(c+dx^n))^p\,dx$

Optimal. Leaf size=21

$$\text{Int}\left((ex)^m (b\sinh(c+dx^n))^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(b*sinh(c+d*x^n))^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m (b\sinh(c+dx^n))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (b\sinh(c+dx^n))^p\,dx = \int (ex)^m (b\sinh(c+dx^n))^p\,dx$$

Mathematica [A] time = 5.41, size = 0, normalized size = 0.00

$$\int (ex)^m (b\sinh(c+dx^n))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(b*Sinh[c + d*x^n])^p, x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m (b\sinh(dx^n+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b \sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int (ex)^m (b \sinh (c + d\, x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^m*(b*sinh(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b \sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c))^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (b \sinh (c + d\, x^n))^p\, (e\, x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x^n))^p*(e*x)^m,x)
[Out] int((b*sinh(c + d*x^n))^p*(e*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sinh (c + dx^n))^p (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*sinh(c+d*x**n))**p,x)
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**m, x)
```

3.75 $\int (ex)^m (a + b\sinh(c + dx^n))^p \, dx$

Optimal. Leaf size=23

$$\text{Int}\left((ex)^m (a + b\sinh(c + dx^n))^p, x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m (a + b\sinh(c + dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Sinh[c + d*x^n])^p,x]
[Out] Defer[Int][(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a + b\sinh(c + dx^n))^p \, dx = \int (ex)^m (a + b\sinh(c + dx^n))^p \, dx$$

Mathematica [A] time = 8.08, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\sinh(c + dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Sinh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Sinh[c + d*x^n])^p, x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m (b\sinh(dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b\sinh(dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\sinh(c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*sinh(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b\sinh(dx^n + c) + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*sinh(d*x^n + c) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,x)^m (a + b\sinh(c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*sinh(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*sinh(c + d*x^n))^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\sinh(c + dx^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*sinh(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*sinh(c + d*x**n))**p, x)
```

3.76 $\int (ex)^{-1+n} (b\sinh(c+dx^n))^p \, dx$

Optimal. Leaf size=94

$$\frac{x^{-n}(ex)^n \cosh(c+dx^n)(b\sinh(c+dx^n))^{p+1} {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};-\sinh^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cosh^2(c+dx^n)}}$$

```
[Out] (e*x)^n*cosh(c+d*x^n)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],-sinh(c+d*x^n)
^2)*(b*sinh(c+d*x^n))^(1+p)/b/d/e/n/(1+p)/(x^n)/(cosh(c+d*x^n)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5322, 5320, 2643}

$$\frac{x^{-n}(ex)^n \cosh(c+dx^n)(b\sinh(c+dx^n))^{p+1} {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};-\sinh^2(dx^n+c)\right)}{bden(p+1)\sqrt{\cosh^2(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*Cosh[c + d*x^n]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, -Sinh
[c + d*x^n]^2]*(b*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n*Sqrt[Cosh[
c + d*x^n]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5322

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
 + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n} (b\sinh(c+dx^n))^p \, dx = \frac{(x^{-n}(ex)^n) \int x^{-1+n} (b\sinh(c+dx^n))^p \, dx}{e}$$
$$= \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int (b\sinh(c+dx))^p \, dx, x, x^n\right)}{en}$$
$$= \frac{x^{-n}(ex)^n \cosh(c+dx^n) \; {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; -\sinh^2(c+dx^n)\right) (b\sinh(c+dx^n}{bden(1+p)\sqrt{\cosh^2(c+dx^n)}}$$

Mathematica [A] time = 0.14, size = 93, normalized size = 0.99

$$-\frac{x^{-n}(ex)^n \sinh(2(c+dx^n)) \left(-\sinh^2(c+dx^n)\right)^{\frac{1}{2}(-p-1)} (b\sinh(c+dx^n))^p \; {}_2F_1\left(\frac{1}{2}, \frac{1-p}{2}; \frac{3}{2}; \cosh^2(dx^n+c)\right)}{2den}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(b*Sinh[c + d*x^n])^p,x]
```

```
[Out] -1/2*((e*x)^n*Hypergeometric2F1[1/2, (1 - p)/2, 3/2, Cosh[c + d*x^n]^2]*(b*
Sinh[c + d*x^n])^p*(-Sinh[c + d*x^n]^2)^((-1 - p)/2)*Sinh[2*(c + d*x^n)])/(
d*e*n*x^n)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((ex)^{n-1} (b\sinh(dx^n+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (b\sinh(dx^n+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} (b \sinh (c + d x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (b \sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b \sinh (c + d x^n))^p (e x)^{n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x^n))^p*(e*x)^(n - 1),x)
[Out] int((b*sinh(c + d*x^n))^p*(e*x)^(n - 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sinh (c + dx^n))^p (ex)^{n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(b*sinh(c+d*x**n))**p,x)
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**(n - 1), x)
```

3.77 $\int (ex)^{-1+2n} (b \sinh (c + dx^n))^p \, dx$

Optimal. Leaf size=39

$$\frac{x^{-2n}(ex)^{2n}\text{Int}\left(x^{2n-1} (b \sinh (c + dx^n))^p , x\right)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^{-1+2n} (b \sinh (c + dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p,x]
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (b \sinh (c + dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right) \int x^{-1+2n} (b \sinh (c + dx^n))^p \, dx}{e}$$

Mathematica [A] time = 5.90, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b \sinh (c + dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Sinh[c + d*x^n])^p, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^{2n-1} (b \sinh (dx^n + c))^p , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2\,n-1} (b\sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b\sinh (c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2\,n-1} (b\sinh (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c))^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int (b\sinh (c + d\,x^n))^p \, (e\,x)^{2\,n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sinh(c + d*x^n))^p*(e*x)^(2*n - 1),x)
[Out] int((b*sinh(c + d*x^n))^p*(e*x)^(2*n - 1), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sinh (c + dx^n))^p \, (ex)^{2n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(b*sinh(c+d*x**n))**p,x)
```

```
[Out] Integral((b*sinh(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

3.78 $\int (ex)^{-1+n} (a + b\sinh(c + dx^n))^p \, dx$

Optimal. Leaf size=150

$$\frac{i\sqrt{2}\,x^{-n}(ex)^n \cosh(c+dx^n)\,(a+b\sinh(c+dx^n))^p \left(\frac{a+b\sinh(c+dx^n)}{a-ib}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-i\sinh(dx^n+c)),\frac{b(1-i\,\mathrm{s}}{}\right.}{den\sqrt{1+i\sinh(c+dx^n)}}$$

```
[Out] I*(e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-I*sinh(c+d*x^n))/(I*a+b),1/2-1/2*I*s
inh(c+d*x^n))*cosh(c+d*x^n)*(a+b*sinh(c+d*x^n))^p*2^(1/2)/d/e/n/(x^n)/(((a+
b*sinh(c+d*x^n))/(a-I*b))^p)/(1+I*sinh(c+d*x^n))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5322, 5320, 2665, 139, 138}

$$\frac{i\sqrt{2}\,x^{-n}(ex)^n \cosh(c+dx^n)\,(a+b\sinh(c+dx^n))^p \left(\frac{a+b\sinh(c+dx^n)}{a-ib}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-i\sinh(dx^n+c)),\frac{b(1-i\,\mathrm{s}}{}\right.}{den\sqrt{1+i\sinh(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] (I*Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - I*Sinh[c + d*x^n])/2, (
b*(1 - I*Sinh[c + d*x^n]))/(I*a + b)]*Cosh[c + d*x^n]*(a + b*Sinh[c + d*x^n
])^p)/(d*e*n*x^n*Sqrt[1 + I*Sinh[c + d*x^n]]*((a + b*Sinh[c + d*x^n])/(a -
I*b))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5322

```
Int[((e_)*(x_))^(m_)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x
_Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
 + b*Sinh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n}\left(a+b\sinh\left(c+dx^n\right)\right)^p\,dx=\frac{\left(x^{-n}(ex)^n\right)\int x^{-1+n}\left(a+b\sinh\left(c+dx^n\right)\right)^p\,dx}{e}$$

$$=\frac{\left(x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int (a+b\sinh(c+dx))^p\,dx,x,x^n\right)}{en}$$

$$=-\frac{\left(ix^{-n}(ex)^n\cosh\left(c+dx^n\right)\right)\operatorname{Subst}\left(\int\frac{(a-ibx)^p}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,i\sinh\left(c+dx^n\right)\right)}{den\sqrt{1-i\sinh\left(c+dx^n\right)}\sqrt{1+i\sinh\left(c+dx^n\right)}}$$

$$=-\frac{\left(ix^{-n}(ex)^n\cosh\left(c+dx^n\right)\left(a+b\sinh\left(c+dx^n\right)\right)^p\left(-\frac{a+b\sinh(c+dx^n)}{-a+ib}\right)^{-p}\right)\mathrm{S}}{den\sqrt{1-i\sinh\left(c+dx^n\right)}\sqrt{1+i\sinh}}$$

$$=\frac{i\sqrt{2}\,x^{-n}(ex)^nF_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}\left(1-i\sinh\left(c+dx^n\right)\right),\frac{b(1-i\sinh(c+dx^n))}{ia+b}\right)\cos}{den\sqrt{1+i\sinh\left(c+dx\right.}}$$

Mathematica [A] time = 0.41, size = 167, normalized size = 1.11

$$\frac{x^{-n}(ex)^n\operatorname{sech}(c+dx^n)\sqrt{\frac{b(1-i\sinh(c+dx^n))}{b+ia}}\sqrt{\frac{b(1+i\sinh(c+dx^n))}{b-ia}}\,(a+b\sinh(c+dx^n))^{p+1}F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{a+b\sinh(c+dx^n)}{a+ib}\right.}{bden(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Sinh[c + d*x^n])/(a + I*b)
, (a + b*Sinh[c + d*x^n])/(a - I*b)]*Sech[c + d*x^n]*Sqrt[(b*(1 - I*Sinh[c
+ d*x^n]))/(I*a + b)]*Sqrt[(b*(1 + I*Sinh[c + d*x^n]))/((-I)*a + b)]*(a + b
*Sinh[c + d*x^n])^(1 + p))/(b*d*e*n*(1 + p)*x^n)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^{n-1}(b\sinh(dx^n+c)+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1}(b\sinh(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n}(a+b\sinh(c+d\,x^n))^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1}(b\sinh(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{n-1}\,\left(a+b\,\sinh\left(c+d\,x^n\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)*(a + b*sinh(c + d*x^n))^p,x)
[Out] int((e*x)^(n - 1)*(a + b*sinh(c + d*x^n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*sinh(c+d*x**n))**p,x)
[Out] Timed out
```

3.79 $\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$

Optimal. Leaf size=41

$$\frac{x^{-2n}(ex)^{2n}\text{Int}\left(x^{2n-1}(a+b\sinh(c+dx^n))^p, x\right)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x])/(e*x^(2
*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(a+b\sinh(c+dx^n))^p \, dx}{e}$$

Mathematica [A] time = 8.45, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a+b\sinh(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sinh[c + d*x^n])^p, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\right)^{2n-1}\left(b\sinh\left(dx^n+c\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2\,n-1}\,(b\sinh\left(dx^{n}+c\right)+a)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

maple [A] time = 0.79, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n}\,(a+b\sinh\left(c+d\,x^{n}\right))^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2\,n-1}\,(b\sinh\left(dx^{n}+c\right)+a)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sinh(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*sinh(d*x^n + c) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(e\,x\right)}^{2\,n-1}\,{\left(a+b\,\mathrm{sinh}\left(c+d\,x^n\right)\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(a + b*sinh(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*sinh(c + d*x^n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sinh(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

3.80 $\int (ex)^m \sinh^3(a+bx^n)\,dx$

Optimal. Leaf size=220

$$-\frac{e^{3a}3^{-\frac{m+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-3bx^n\right)}{8en}+\frac{3e^a(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-bx^n\right)}{8en}-\frac{3e^{-a}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\Gamma\left(\right.}{8en}$$

```
[Out] -1/8*exp(3*a)*(e*x)^(1+m)*GAMMA((1+m)/n,-3*b*x^n)/(3^((1+m)/n))/e/n/((-b*x^
n)^((1+m)/n))+3/8*exp(a)*(e*x)^(1+m)*GAMMA((1+m)/n,-b*x^n)/e/n/((-b*x^n)^((
1+m)/n))-3/8*(e*x)^(1+m)*GAMMA((1+m)/n,b*x^n)/e/exp(a)/n/((b*x^n)^((1+m)/n)
)+1/8*(e*x)^(1+m)*GAMMA((1+m)/n,3*b*x^n)/(3^((1+m)/n))/e/exp(3*a)/n/((b*x^n
)^((1+m)/n))
```

Rubi [A] time = 0.23, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5362, 5360, 2218}

$$-\frac{e^{3a}3^{-\frac{m+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-3bx^n\right)}{8en}+\frac{3e^a(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{8en}-\frac{3e^{-a}(ex)^{m}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^n]^3,x]
```

```
[Out] -(E^(3*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(8*3^((1 + m)/n)*e*n*(-
(b*x^n))^((1 + m)/n)) + (3*E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(8
*e*n*(-(b*x^n))^((1 + m)/n)) - (3*(e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(8
*e*E^a*n*(b*x^n)^((1 + m)/n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])/(
8*3^((1 + m)/n)*e*E^(3*a)*n*(b*x^n)^((1 + m)/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^3(a+bx^n)\,dx &= \int \left(-\frac{3}{4}(ex)^m \sinh(a+bx^n)+\frac{1}{4}(ex)^m \sinh(3a+3bx^n)\right)dx\\ &= \frac{1}{4}\int (ex)^m \sinh(3a+3bx^n)\,dx-\frac{3}{4}\int (ex)^m \sinh(a+bx^n)\,dx\\ &= -\left(\frac{1}{8}\int e^{-3a-3bx^n}(ex)^m\,dx\right)+\frac{1}{8}\int e^{3a+3bx^n}(ex)^m\,dx+\frac{3}{8}\int e^{-a-bx^n}(ex)^m\,dx-\frac{3}{8}\int e^{a\cdots}\\ &= -\frac{3^{-\frac{1+m}{n}}e^{3a}(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-3bx^n\right)}{8en}+\frac{3e^a(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx\cdots\right.}{8en}\end{aligned}$$

Mathematica [A] time = 2.15, size = 185, normalized size = 0.84

$$\frac{e^{-3a}x3^{-\frac{m+1}{n}}(ex)^m\left(-b^2x^{2n}\right)^{-\frac{m+1}{n}}\left((-bx^n)^{\frac{m+1}{n}}\left(\Gamma\left(\frac{m+1}{n},3bx^n\right)-e^{2a}3^{\frac{m+n+1}{n}}\Gamma\left(\frac{m+1}{n},bx^n\right)\right)-e^{6a}(bx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-3bx^n\right)\cdots\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n]^3,x]
```

```
[Out] (x*(e*x)^m*(-(E^(6*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -3*b*x^n]) + 3^(
(1 + m + n)/n)*E^(4*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)] + (-(
b*x^n))^((1 + m)/n)*(-(3^((1 + m + n)/n)*E^(2*a)*Gamma[(1 + m)/n, b*x^n]) +
 Gamma[(1 + m)/n, 3*b*x^n])))/(8*3^((1 + m)/n)*E^(3*a)*n*(-(b^2*x^(2*n)))^(
(1 + m)/n))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \sinh(bx^n+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh (bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate((e*x)^m*sinh(b*x^n + a)^3, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (ex)^m \left(\sinh^3 (a + b\,x^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b*x^n)^3,x)
[Out] int((e*x)^m*sinh(a+b*x^n)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh (bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate((e*x)^m*sinh(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh (a + b\,x^n)^3\,(e\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^3*(e*x)^m,x)
[Out] int(sinh(a + b*x^n)^3*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh^3 (a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b*x**n)**3,x)
[Out] Integral((e*x)**m*sinh(a + b*x**n)**3, x)
```

3.81 $\int (ex)^m \sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=143

$$-\frac{e^{2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2bx^n\right)}{en}-\frac{e^{-2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2bx^n\right)}{en}-\frac{(ex)^{m+1}}{2e(m+1)}$$

```
[Out] -1/2*(e*x)^(1+m)/e/(1+m)-exp(2*a)*(e*x)^(1+m)*GAMMA((1+m)/n,-2*b*x^n)/(2^((
1+m+2*n)/n))/e/n/((-b*x^n)^((1+m)/n))-(e*x)^(1+m)*GAMMA((1+m)/n,2*b*x^n)/(2
^((1+m+2*n)/n))/e/exp(2*a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5362, 5361, 2218}

$$-\frac{e^{2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2bx^n\right)}{en}-\frac{e^{-2a}2^{-\frac{m+2n+1}{n}}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2bx^n\right)}{en}-\frac{(}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^n]^2,x]
```

```
[Out] -(e*x)^(1 + m)/(2*e*(1 + m)) - (E^(2*a)*(e*x)^(1 + m)*Gamma[(1 + m)/n, -2*b
*x^n])/(2^((1 + m + 2*n)/n)*e*n*(-(b*x^n))^((1 + m)/n)) - ((e*x)^(1 + m)*Ga
mma[(1 + m)/n, 2*b*x^n])/(2^((1 + m + 2*n)/n)*e*E^(2*a)*n*(b*x^n)^((1 + m)/
n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5361

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (ex)^m \sinh^2\left(a+bx^n\right) dx &= \int \left(-\frac{1}{2}(ex)^m + \frac{1}{2}(ex)^m \cosh\left(2a+2bx^n\right)\right) dx\\ &= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{2}\int (ex)^m \cosh\left(2a+2bx^n\right) dx\\ &= -\frac{(ex)^{1+m}}{2e(1+m)} + \frac{1}{4}\int e^{-2a-2bx^n}(ex)^m\, dx + \frac{1}{4}\int e^{2a+2bx^n}(ex)^m\, dx\\ &= -\frac{(ex)^{1+m}}{2e(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a}(ex)^{1+m}\left(-bx^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2bx^n\right)}{en} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a}(ex)^{1\cdot}}{}\end{aligned}$$

Mathematica [A] time = 2.00, size = 117, normalized size = 0.82

$$-\frac{x(ex)^m\left(e^{2a}(m+1)2^{-\frac{m+1}{n}}\left(-bx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2bx^n\right)+e^{-2a}(m+1)2^{-\frac{m+1}{n}}\left(bx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2bx^n\right)+2n\right)}{4(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x*(e*x)^m*(2*n + (E^(2*a)*(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^((1
+ m)/n)*(-(b*x^n))^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, 2*b*x^n])/(2^((
1 + m)/n)*E^(2*a)*(b*x^n)^((1 + m)/n))))/((1 + m)*n)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \sinh\left(bx^n+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh\left(bx^n+a\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a)^2, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (ex)^{m} \left(\sinh^{2}\left(a+b\,x^{n}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b*x^n)^2,x)
```

```
[Out] int((e*x)^m*sinh(a+b*x^n)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,e^{m}\int e^{\left(2\,bx^{n}+m\log(x)+2\,a\right)}\,dx+\frac{1}{4}\,e^{m}\int e^{\left(-2\,bx^{n}+m\log(x)-2\,a\right)}\,dx-\frac{(ex)^{m+1}}{2\,e(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/4*e^m*integrate(e^(2*b*x^n + m*log(x) + 2*a), x) + 1/4*e^m*integrate(e^(-
2*b*x^n + m*log(x) - 2*a), x) - 1/2*(e*x)^(m + 1)/(e*(m + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(a+b\,x^{n}\right)^{2}\,(e\,x)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^2*(e*x)^m,x)
```

```
[Out] int(sinh(a + b*x^n)^2*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{m}\sinh^{2}\left(a+bx^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**n)**2, x)
```

3.82 $\int (ex)^m \sinh(a+bx^n)\,dx$

Optimal. Leaf size=99

$$\frac{e^{-a}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},bx^n\right)}{2en}-\frac{e^{a}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-bx^n\right)}{2en}$$

```
[Out] -1/2*exp(a)*(e*x)^(1+m)*GAMMA((1+m)/n,-b*x^n)/e/n/((-b*x^n)^((1+m)/n))+1/2*
(e*x)^(1+m)*GAMMA((1+m)/n,b*x^n)/e/exp(a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5360, 2218}

$$\frac{e^{-a}(ex)^{m+1}(bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},bx^n\right)}{2en}-\frac{e^{a}(ex)^{m+1}(-bx^n)^{-\frac{m+1}{n}}\,\text{Gamma}\left(\frac{m+1}{n},-bx^n\right)}{2en}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sinh[a + b*x^n],x]
```

```
[Out] -(E^a*(e*x)^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(2*e*n*(-(b*x^n))^((1 + m)/
n)) + ((e*x)^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(2*e*E^a*n*(b*x^n)^((1 + m)/n
))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5360

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2
, Int[(e*x)^m*E^(c + d*x^n), x], x] - Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n),
 x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int (ex)^m \sinh(a+bx^n)\,dx = -\left(\frac{1}{2}\int e^{-a-bx^n}(ex)^m\,dx\right) + \frac{1}{2}\int e^{a+bx^n}(ex)^m\,dx$$
$$= -\frac{e^a(ex)^{1+m}(-bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-bx^n\right)}{2en} + \frac{e^{-a}(ex)^{1+m}(bx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},bx^n\right)}{2en}$$

Mathematica [A] time = 0.18, size = 102, normalized size = 1.03

$$-\frac{x(ex)^m\left(-b^2x^{2n}\right)^{-\frac{m+1}{n}}\left((\sinh(a)+\cosh(a))(bx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-bx^n\right)-(\cosh(a)-\sinh(a))(-bx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},bx^n\right)\right.}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sinh[a + b*x^n],x]
```

```
[Out] -1/2*(x*(e*x)^m*(-((-(b*x^n))^((1 + m)/n)*Gamma[(1 + m)/n, b*x^n]*(Cosh[a]
- Sinh[a])) + (b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)]*(Cosh[a] + Sin
h[a])))/(n*(-(b^2*x^(2*n)))^((1 + m)/n))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m \sinh(bx^n+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*sinh(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh(bx^n+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a), x)
```

maple [C] time = 0.27, size = 115, normalized size = 1.16

$$\frac{(ex)^m x\,\text{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],\frac{x^{2n}b^2}{4}\right)\sinh(a)}{1+m}+\frac{(ex)^m x^{n+1}b\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}\right.\right.}{m+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*sinh(a+b*x^n),x)
```

```
[Out] (e*x)^m/(1+m)*x*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],1/4*x^(2*n)
*b^2)*sinh(a)+(e*x)^m/(m+n+1)*x^(n+1)*b*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,
3/2+1/2/n*m+1/2/n],1/4*x^(2*n)*b^2)*cosh(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh (bx^n + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m*sinh(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh (a + b\,x^n)\,(e\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)*(e*x)^m,x)
```

```
[Out] int(sinh(a + b*x^n)*(e*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \sinh (a + bx^n)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*sinh(a+b*x**n),x)
```

```
[Out] Integral((e*x)**m*sinh(a + b*x**n), x)
```

3.83 $\int (ex)^m \mathrm{csch}^2(a+bx^n)\,dx$

Optimal. Leaf size=28

$$x^{-m}(ex)^m \mathrm{Int}\left(x^m \mathrm{csch}^2(a+bx^n), x\right)$$

```
[Out] (e*x)^m*Unintegrable(x^m*csch(a+b*x^n)^2,x)/(x^m)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m \mathrm{csch}^2(a+bx^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*Csch[a + b*x^n]^2,x]
[Out] ((e*x)^m*Defer[Int][x^m*Csch[a + b*x^n]^2, x])/x^m
```

Rubi steps

$$\int (ex)^m \mathrm{csch}^2(a+bx^n)\,dx = \left(x^{-m}(ex)^m\right)\int x^m \mathrm{csch}^2(a+bx^n)\,dx$$

Mathematica [A] time = 24.34, size = 0, normalized size = 0.00

$$\int (ex)^m \mathrm{csch}^2(a+bx^n)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*Csch[a + b*x^n]^2,x]
[Out] Integrate[(e*x)^m*Csch[a + b*x^n]^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex)^m}{\sinh(bx^n+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="fricas")
[Out] integral((e*x)^m/sinh(b*x^n + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sinh(b*x^n + a)^2, x)
```

maple [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh(a + b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a+b*x^n)^2,x)
```

```
[Out] int((e*x)^m/sinh(a+b*x^n)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-4\,e^m(m-n+1)\int \frac{x^m}{4\left(bnx^n + bne^{\left(bx^n+n\log(x)+a\right)}\right)}\,dx + 4\,e^m(m-n+1)\int -\frac{x^m}{4\left(bnx^n - bne^{\left(bx^n+n\log(x)+a\right)}\right)}\,dx + \frac{}{bnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -4*e^m*(m - n + 1)*integrate(1/4*x^m/(b*n*x^n + b*n*e^(b*x^n + n*log(x) + a
)), x) + 4*e^m*(m - n + 1)*integrate(-1/4*x^m/(b*n*x^n - b*n*e^(b*x^n + n*l
og(x) + a)), x) + 2*e^m*x*x^m/(b*n*x^n - b*n*e^(2*b*x^n + n*log(x) + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,x)^m}{\sinh(a + b\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/sinh(a + b*x^n)^2,x)
```

```
[Out] int((e*x)^m/sinh(a + b*x^n)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sinh^2(a + bx^n)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/sinh(a+b*x**n)**2,x)
```

```
[Out] Integral((e*x)**m/sinh(a + b*x**n)**2, x)
```

3.84 $\int x^{-1-n} \sinh(a + bx^n)\, dx$

Optimal. Leaf size=45

$$\frac{b\cosh(a)\text{Chi}(bx^n)}{n} + \frac{b\sinh(a)\text{Shi}(bx^n)}{n} - \frac{x^{-n}\sinh(a + bx^n)}{n}$$

```
[Out] b*Chi(b*x^n)*cosh(a)/n+b*Shi(b*x^n)*sinh(a)/n-sinh(a+b*x^n)/n/(x^n)
```

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5320, 3297, 3303, 3298, 3301}

$$\frac{b\cosh(a)\text{Chi}(bx^n)}{n} + \frac{b\sinh(a)\text{Shi}(bx^n)}{n} - \frac{x^{-n}\sinh(a + bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n],x]
```

```
[Out] (b*Cosh[a]*CoshIntegral[b*x^n])/n - Sinh[a + b*x^n]/(n*x^n) + (b*Sinh[a]*Si
nhIntegral[b*x^n])/n
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx = \frac{\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-n}\sinh\left(a+bx^n\right)}{n} + \frac{b\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-n}\sinh\left(a+bx^n\right)}{n} + \frac{(b\cosh(a))\,\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx, x, x^n\right)}{n} + \frac{(b\sinh(a))\,\text{Subst}\left(\right.}{n}$$

$$= \frac{b\cosh(a)\text{Chi}\left(bx^n\right)}{n} - \frac{x^{-n}\sinh\left(a+bx^n\right)}{n} + \frac{b\sinh(a)\text{Shi}\left(bx^n\right)}{n}$$

Mathematica [A] time = 0.07, size = 46, normalized size = 1.02

$$\frac{x^{-n}\left(b\cosh(a)x^n\text{Chi}\left(bx^n\right) + b\sinh(a)x^n\text{Shi}\left(bx^n\right) - \sinh\left(a+bx^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n],x]
```

```
[Out] (b*x^n*Cosh[a]*CoshIntegral[b*x^n] - Sinh[a + b*x^n] + b*x^n*Sinh[a]*SinhIn
tegral[b*x^n])/(n*x^n)
```

fricas [B] time = 0.46, size = 139, normalized size = 3.09

$$\frac{\left((b\cosh(a) + b\sinh(a))\cosh\left(n\log(x)\right) + (b\cosh(a) + b\sinh(a))\sinh\left(n\log(x)\right)\right)\text{Ei}\left(b\cosh\left(n\log(x)\right) + b\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh
(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + ((b*cosh(a) - b*sinh(
a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*l
og(x)) - b*sinh(n*log(x))) - 2*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a
))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1} \sinh\left(bx^{n} + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a), x)
```

maple [A] time = 0.05, size = 74, normalized size = 1.64

$$\frac{e^{-a-b\,x^{n}}x^{-n}}{2n}-\frac{b\,e^{-a}\,\mathrm{Ei}\left(1,b\,x^{n}\right)}{2n}-\frac{e^{a+b\,x^{n}}x^{-n}}{2n}-\frac{b\,e^{a}\,\mathrm{Ei}\left(1,-b\,x^{n}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sinh(a+b*x^n),x)
```

```
[Out] 1/2/n*exp(-a-b*x^n)/(x^n)-1/2/n*b*exp(-a)*Ei(1,b*x^n)-1/2*exp(a+b*x^n)/(x^n
)/n-1/2/n*b*exp(a)*Ei(1,-b*x^n)
```

maxima [A] time = 0.41, size = 34, normalized size = 0.76

$$\frac{be^{(-a)}\Gamma\left(-1,bx^{n}\right)}{2\,n}+\frac{be^{a}\Gamma\left(-1,-bx^{n}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*e^(-a)*gamma(-1, b*x^n)/n + 1/2*b*e^a*gamma(-1, -b*x^n)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(a+b\,x^{n}\right)}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)/x^(n + 1),x)
[Out] int(sinh(a + b*x^n)/x^(n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1} \sinh\left(a + bx^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n),x)
[Out] Integral(x**(-n - 1)*sinh(a + b*x**n), x)
```

3.85 $\int x^{-1-n}\sinh^2(a+bx^n)\,dx$

Optimal. Leaf size=67

$$\frac{b\sinh(2a)\mathrm{Chi}\left(2bx^n\right)}{n}+\frac{b\cosh(2a)\mathrm{Shi}\left(2bx^n\right)}{n}-\frac{x^{-n}\cosh\left(2\left(a+bx^n\right)\right)}{2n}+\frac{x^{-n}}{2n}$$

```
[Out] 1/2/n/(x^n)-1/2*cosh(2*a+2*b*x^n)/n/(x^n)+b*cosh(2*a)*Shi(2*b*x^n)/n+b*Chi(
2*b*x^n)*sinh(2*a)/n
```

Rubi [A] time = 0.12, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5362, 5321, 3297, 3303, 3298, 3301}

$$\frac{b\sinh(2a)\mathrm{Chi}\left(2bx^n\right)}{n}+\frac{b\cosh(2a)\mathrm{Shi}\left(2bx^n\right)}{n}-\frac{x^{-n}\cosh\left(2\left(a+bx^n\right)\right)}{2n}+\frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n]^2,x]
```

```
[Out] 1/(2*n*x^n) - Cosh[2*(a + b*x^n)]/(2*n*x^n) + (b*CoshIntegral[2*b*x^n]*Sinh
[2*a])/n + (b*Cosh[2*a]*SinhIntegral[2*b*x^n])/n
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5321

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\sinh^2(a+bx^n)\,dx &= \int\left(-\frac{1}{2}x^{-1-n}+\frac{1}{2}x^{-1-n}\cosh(2a+2bx^n)\right)dx\\
&= \frac{x^{-n}}{2n}+\frac{1}{2}\int x^{-1-n}\cosh(2a+2bx^n)\,dx\\
&= \frac{x^{-n}}{2n}+\frac{\mathrm{Subst}\left(\int\frac{\cosh(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}\\
&= \frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{b\,\mathrm{Subst}\left(\int\frac{\sinh(2a+2bx)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{(b\cosh(2a))\,\mathrm{Subst}\left(\int\frac{\sinh(2bx)}{x}\,dx,x,x^n\right)}{n}+\frac{(b\sinh(}{}\\
&= \frac{x^{-n}}{2n}-\frac{x^{-n}\cosh(2(a+bx^n))}{2n}+\frac{b\mathrm{Chi}(2bx^n)\sinh(2a)}{n}+\frac{b\cosh(2a)\mathrm{Shi}(2bx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 54, normalized size = 0.81

$$\frac{x^{-n}\left(b\sinh(2a)x^n\mathrm{Chi}(2bx^n)+b\cosh(2a)x^n\mathrm{Shi}(2bx^n)-\sinh^2(a+bx^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n]^2,x]
```

```
[Out] (b*x^n*CoshIntegral[2*b*x^n]*Sinh[2*a] - Sinh[a + b*x^n]^2 + b*x^n*Cosh[2*a
]*SinhIntegral[2*b*x^n])/(n*x^n)
```

fricas [B] time = 0.63, size = 182, normalized size = 2.72

$$\left((b\cosh(2a) + b\sinh(2a))\cosh\left(n\log(x)\right) + (b\cosh(2a) + b\sinh(2a))\sinh\left(n\log(x)\right)\right)\mathrm{Ei}\left(2b\cosh\left(n\log(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(2*a) + b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) + b*sinh(2*
a))*sinh(n*log(x)))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) - ((b*cosh(
2*a) - b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) - b*sinh(2*a))*sinh(n*log
(x)))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - cosh(b*cosh(n*log(x))
+ b*sinh(n*log(x)) + a)^2 - sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2
 + 1)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\sinh\left(bx^n + a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a)^2, x)
```

maple [A] time = 0.12, size = 90, normalized size = 1.34

$$\frac{x^{-n}}{2n} - \frac{\mathrm{e}^{-2a-2b\,x^n}x^{-n}}{4n} + \frac{b\,\mathrm{e}^{-2a}\,\mathrm{Ei}\left(1, 2b\,x^n\right)}{2n} - \frac{x^{-n}\mathrm{e}^{2a+2b\,x^n}}{4n} - \frac{b\,\mathrm{e}^{2a}\,\mathrm{Ei}\left(1, -2b\,x^n\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sinh(a+b*x^n)^2,x)
```

```
[Out] 1/2/n/(x^n)-1/4/n*exp(-2*a-2*b*x^n)/(x^n)+1/2/n*b*exp(-2*a)*Ei(1,2*b*x^n)-1
/4/(x^n)*exp(2*a+2*b*x^n)/n-1/2/n*b*exp(2*a)*Ei(1,-2*b*x^n)
```

maxima [A] time = 0.43, size = 47, normalized size = 0.70

$$-\frac{be^{(-2a)}\Gamma\left(-1, 2bx^n\right)}{2n} + \frac{be^{(2a)}\Gamma\left(-1, -2bx^n\right)}{2n} + \frac{1}{2nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-2*a)*gamma(-1, 2*b*x^n)/n + 1/2*b*e^(2*a)*gamma(-1, -2*b*x^n)/n
+ 1/2/(n*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+b\,x^n\right)^2}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^2/x^(n + 1),x)
```

```
[Out] int(sinh(a + b*x^n)^2/x^(n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\sinh^2\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-n - 1)*sinh(a + b*x**n)**2, x)
```

3.86 $\int x^{-1-n}\sinh^3(a+bx^n)\,dx$

Optimal. Leaf size=113

$$-\frac{3b\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{3b\cosh(3a)\text{Chi}(3bx^n)}{4n}-\frac{3b\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{3b\sinh(3a)\text{Shi}(3bx^n)}{4n}+\frac{3x^{-n}\sinh(a+bx}{4n}$$

```
[Out] -3/4*b*Chi(b*x^n)*cosh(a)/n+3/4*b*Chi(3*b*x^n)*cosh(3*a)/n-3/4*b*Shi(b*x^n)
*sinh(a)/n+3/4*b*Shi(3*b*x^n)*sinh(3*a)/n+3/4*sinh(a+b*x^n)/n/(x^n)-1/4*sin
h(3*a+3*b*x^n)/n/(x^n)
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5362, 5320, 3297, 3303, 3298, 3301}

$$-\frac{3b\cosh(a)\text{Chi}(bx^n)}{4n}+\frac{3b\cosh(3a)\text{Chi}(3bx^n)}{4n}-\frac{3b\sinh(a)\text{Shi}(bx^n)}{4n}+\frac{3b\sinh(3a)\text{Shi}(3bx^n)}{4n}+\frac{3x^{-n}\sinh(a+bx}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Sinh[a + b*x^n]^3,x]
```

```
[Out] (-3*b*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*CoshIntegral[3*b*
x^n])/(4*n) + (3*Sinh[a + b*x^n])/(4*n*x^n) - Sinh[3*(a + b*x^n)]/(4*n*x^n)
 - (3*b*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Sinh[3*a]*SinhIntegral[3*
b*x^n])/(4*n)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5362

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Sinh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n}\sinh^3\left(a+bx^n\right)\,dx &= \int \left(-\frac{3}{4}x^{-1-n}\sinh\left(a+bx^n\right)+\frac{1}{4}x^{-1-n}\sinh\left(3a+3bx^n\right)\right)dx\\
&= \frac{1}{4}\int x^{-1-n}\sinh\left(3a+3bx^n\right)\,dx-\frac{3}{4}\int x^{-1-n}\sinh\left(a+bx^n\right)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(3a+3bx)}{x^2}\,dx,x,x^n\right)}{4n}-\frac{3\,\text{Subst}\left(\int \frac{\sinh(a+bx)}{x^2}\,dx,x,x^n\right)}{4n}\\
&= \frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\sinh\left(3\left(a+bx^n\right)\right)}{4n}-\frac{(3b)\,\text{Subst}\left(\int \frac{\cosh(a+bx)}{x}\,dx,x,x^n\right)}{4n}+\\
&= \frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\sinh\left(3\left(a+bx^n\right)\right)}{4n}-\frac{(3b\cosh(a))\,\text{Subst}\left(\int \frac{\cosh(bx)}{x}\,dx,x,\right.}{4n}\\
&= -\frac{3b\cosh(a)\text{Chi}\left(bx^n\right)}{4n}+\frac{3b\cosh(3a)\text{Chi}\left(3bx^n\right)}{4n}+\frac{3x^{-n}\sinh\left(a+bx^n\right)}{4n}-\frac{x^{-n}\sinh}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 95, normalized size = 0.84

$$-\frac{x^{-n}\left(3b\cosh(a)x^n\text{Chi}\left(bx^n\right)-3b\cosh(3a)x^n\text{Chi}\left(3bx^n\right)+3b\sinh(a)x^n\text{Shi}\left(bx^n\right)-3b\sinh(3a)x^n\text{Shi}\left(3bx^n\right)-3\,\text{s}\right.}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Sinh[a + b*x^n]^3,x]
```

```
[Out] -1/4*(3*b*x^n*Cosh[a]*CoshIntegral[b*x^n] - 3*b*x^n*Cosh[3*a]*CoshIntegral[
3*b*x^n] - 3*Sinh[a + b*x^n] + Sinh[3*(a + b*x^n)] + 3*b*x^n*Sinh[a]*SinhIn
tegral[b*x^n] - 3*b*x^n*Sinh[3*a]*SinhIntegral[3*b*x^n])/(n*x^n)
```

fricas [B] time = 0.54, size = 303, normalized size = 2.68

$$-\frac{2\sinh\left(b\cosh\left(n\log(x)\right)+b\sinh\left(n\log(x)\right)+a\right)^{3}-3\left(\left(b\cosh\left(3a\right)+b\sinh\left(3a\right)\right)\cosh\left(n\log(x)\right)+\left(b\cosh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^3 - 3*((b*cosh(3*a) +
 b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) + b*sinh(3*a))*sinh(n*log(x)))*
Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) + 3*((b*cosh(a) + b*sinh(a))*co
sh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*log(x)))*Ei(b*cosh(n*log(x))
+ b*sinh(n*log(x))) + 3*((b*cosh(a) - b*sinh(a))*cosh(n*log(x)) + (b*cosh(a
) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) - 3
*((b*cosh(3*a) - b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) - b*sinh(3*a))*
sinh(n*log(x)))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))) + 6*(cosh(b*co
sh(n*log(x)) + b*sinh(n*log(x)) + a)^2 - 1)*sinh(b*cosh(n*log(x)) + b*sinh(
n*log(x)) + a))/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\sinh\left(bx^{n}+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*sinh(b*x^n + a)^3, x)
```

maple [A] time = 0.17, size = 152, normalized size = 1.35

$$\frac{{\rm e}^{-3a-3b\,x^{n}}x^{-n}}{8n}-\frac{3b\,{\rm e}^{-3a}\,\mathrm{Ei}\left(1,3b\,x^{n}\right)}{8n}-\frac{3\,{\rm e}^{-a-b\,x^{n}}x^{-n}}{8n}+\frac{3b\,{\rm e}^{-a}\,\mathrm{Ei}\left(1,b\,x^{n}\right)}{8n}-\frac{x^{-n}{\rm e}^{3a+3b\,x^{n}}}{8n}-\frac{3b\,{\rm e}^{3a}\,\mathrm{Ei}\left(1,-3b\,x^{n}\right)}{8n}+\frac{3\,{\rm e}^{a+b\,x}}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*sinh(a+b*x^n)^3,x)
```

```
[Out] 1/8/n*exp(-3*a-3*b*x^n)/(x^n)-3/8/n*b*exp(-3*a)*Ei(1,3*b*x^n)-3/8/n*exp(-a-
b*x^n)/(x^n)+3/8/n*b*exp(-a)*Ei(1,b*x^n)-1/8/(x^n)*exp(3*a+3*b*x^n)/n-3/8/n
*b*exp(3*a)*Ei(1,-3*b*x^n)+3/8*exp(a+b*x^n)/(x^n)/n+3/8/n*b*exp(a)*Ei(1,-b*
x^n)
```

maxima [A] time = 0.48, size = 70, normalized size = 0.62

$$\frac{3\,be^{(-3\,a)}\Gamma\left(-1,3\,bx^n\right)}{8\,n}-\frac{3\,be^{(-a)}\Gamma\left(-1,bx^n\right)}{8\,n}-\frac{3\,be^{a}\Gamma\left(-1,-bx^n\right)}{8\,n}+\frac{3\,be^{(3\,a)}\Gamma\left(-1,-3\,bx^n\right)}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*sinh(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] 3/8*b*e^(-3*a)*gamma(-1, 3*b*x^n)/n - 3/8*b*e^(-a)*gamma(-1, b*x^n)/n - 3/8
*b*e^a*gamma(-1, -b*x^n)/n + 3/8*b*e^(3*a)*gamma(-1, -3*b*x^n)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+b\,x^n\right)^3}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x^n)^3/x^(n + 1),x)
```

```
[Out] int(sinh(a + b*x^n)^3/x^(n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*sinh(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

3.87 $\int x^{-1+\frac{n}{2}} \sinh(a+bx^n)\, dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\pi}\, e^a \operatorname{erfi}\left(\sqrt{b}\, x^{n/2}\right)}{2\sqrt{b}\, n}-\frac{\sqrt{\pi}\, e^{-a} \operatorname{erf}\left(\sqrt{b}\, x^{n/2}\right)}{2\sqrt{b}\, n}$$

```
[Out] -1/2*erf(x^(1/2*n)*b^(1/2))*Pi^(1/2)/exp(a)/n/b^(1/2)+1/2*exp(a)*erfi(x^(1/
2*n)*b^(1/2))*Pi^(1/2)/n/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5356, 5298, 2204, 2205}

$$\frac{\sqrt{\pi}\, e^a \operatorname{Erfi}\left(\sqrt{b}\, x^{n/2}\right)}{2\sqrt{b}\, n}-\frac{\sqrt{\pi}\, e^{-a} \operatorname{Erf}\left(\sqrt{b}\, x^{n/2}\right)}{2\sqrt{b}\, n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)*Sinh[a + b*x^n],x]
```

```
[Out] -(Sqrt[Pi]*Erf[Sqrt[b]*x^(n/2)])/(2*Sqrt[b]*E^a*n) + (E^a*Sqrt[Pi]*Erfi[Sqr
t[b]*x^(n/2)])/(2*Sqrt[b]*n)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5356

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/(m + 1), Subst[Int[(a + b*Sinh[c + d*x^Simplify[n/(m + 1)]])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegerQ[p] && N
eQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int x^{-1+\frac{n}{2}}\sinh\left(a+bx^n\right)\,dx &= \frac{2\,\text{Subst}\left(\int \sinh\left(a+bx^2\right)\,dx,x,x^{n/2}\right)}{n}\\ &= -\frac{\text{Subst}\left(\int e^{-a-bx^2}\,dx,x,x^{n/2}\right)}{n}+\frac{\text{Subst}\left(\int e^{a+bx^2}\,dx,x,x^{n/2}\right)}{n}\\ &= -\frac{e^{-a}\sqrt{\pi}\,\text{erf}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}+\frac{e^{a}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}\,x^{n/2}\right)}{2\sqrt{b}\,n}\end{aligned}$$

Mathematica [A] time = 1.58, size = 60, normalized size = 0.85

$$\frac{\sqrt{\pi}\,\left((\sinh(a)-\cosh(a))\text{erf}\left(\sqrt{b}\,x^{n/2}\right)+(\sinh(a)+\cosh(a))\text{erfi}\left(\sqrt{b}\,x^{n/2}\right)\right)}{2\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)*Sinh[a + b*x^n],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x^(n/2)]*(-Cosh[a] + Sinh[a]) + Erfi[Sqrt[b]*x^(n/2)
]*(Cosh[a] + Sinh[a])))/(2*Sqrt[b]*n)
```

fricas [A] time = 0.72, size = 97, normalized size = 1.37

$$-\frac{\sqrt{\pi}\,\sqrt{-b}\,(\cosh(a)+\sinh(a))\,\text{erf}\left(\sqrt{-b}\,x\cosh\left(\frac{1}{2}\,(n-2)\log(x)\right)+\sqrt{-b}\,x\sinh\left(\frac{1}{2}\,(n-2)\log(x)\right)\right)+\sqrt{\pi}\,\sqrt{b}\,(\cos}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x*cosh(1/2*(n - 2)
*log(x)) + sqrt(-b)*x*sinh(1/2*(n - 2)*log(x))) + sqrt(pi)*sqrt(b)*(cosh(a)
 - sinh(a))*erf(sqrt(b)*x*cosh(1/2*(n - 2)*log(x)) + sqrt(b)*x*sinh(1/2*(n
- 2)*log(x))))/(b*n)
```

giac [A] time = 0.15, size = 53, normalized size = 0.75

$$\frac{\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,\sqrt{x^{n}}\right)e^{(-a)}}{\sqrt{b}} - \frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{-b}\,\sqrt{x^{n}}\right)e^{a}}{\sqrt{-b}}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(pi)*erf(-sqrt(b)*sqrt(x^n))*e^(-a)/sqrt(b) - sqrt(pi)*erf(-sqrt(-
b)*sqrt(x^n))*e^a/sqrt(-b))/n
```

maple [A] time = 0.08, size = 54, normalized size = 0.76

$$-\frac{e^{-a}\sqrt{\pi}\ \operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2n\sqrt{b}} + \frac{e^{a}\sqrt{\pi}\ \operatorname{erf}\left(\sqrt{-b}\ x^{\frac{n}{2}}\right)}{2n\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)*sinh(a+b*x^n),x)
```

```
[Out] -1/2/n*exp(-a)*Pi^(1/2)/b^(1/2)*erf(x^(1/2*n)*b^(1/2))+1/2/n*exp(a)*Pi^(1/2
)/(-b)^(1/2)*erf((-b)^(1/2)*x^(1/2*n))
```

maxima [A] time = 0.40, size = 69, normalized size = 0.97

$$-\frac{\sqrt{\pi}\,x^{\frac{1}{2}\,n}\left(\operatorname{erf}\left(\sqrt{bx^{n}}\right)-1\right)e^{(-a)}}{2\,\sqrt{bx^{n}}\,n} + \frac{\sqrt{\pi}\,x^{\frac{1}{2}\,n}\left(\operatorname{erf}\left(\sqrt{-bx^{n}}\right)-1\right)e^{a}}{2\,\sqrt{-bx^{n}}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)*sinh(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(b*x^n)) - 1)*e^(-a)/(sqrt(b*x^n)*n) + 1/2
*sqrt(pi)*x^(1/2*n)*(erf(sqrt(-b*x^n)) - 1)*e^a/(sqrt(-b*x^n)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{\frac{n}{2}-1}\sinh\left(a+b\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)*sinh(a + b*x^n),x)
```

```
[Out] int(x^(n/2 - 1)*sinh(a + b*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{n}{2}-1} \sinh\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)*sinh(a+b*x**n),x)
```

```
[Out] Integral(x**(n/2 - 1)*sinh(a + b*x**n), x)
```

3.88 $\int x^2 \sinh\left((a+bx)^2\right) dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{\pi}\, a^2 \text{erf}(a+bx)}{4b^3}+\frac{\sqrt{\pi}\, a^2 \text{erfi}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\, \text{erf}(a+bx)}{8b^3}-\frac{\sqrt{\pi}\, \text{erfi}(a+bx)}{8b^3}-\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left(\right.}{2b^3}$$

```
[Out] -a*cosh((b*x+a)^2)/b^3+1/2*(b*x+a)*cosh((b*x+a)^2)/b^3-1/8*erf(b*x+a)*Pi^(1
/2)/b^3-1/4*a^2*erf(b*x+a)*Pi^(1/2)/b^3-1/8*erfi(b*x+a)*Pi^(1/2)/b^3+1/4*a^
2*erfi(b*x+a)*Pi^(1/2)/b^3
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5364, 6742, 5298, 2204, 2205, 5320, 2638, 5324, 5299}

$$-\frac{\sqrt{\pi}\, a^2 \text{Erf}(a+bx)}{4b^3}+\frac{\sqrt{\pi}\, a^2 \text{Erfi}(a+bx)}{4b^3}-\frac{\sqrt{\pi}\, \text{Erf}(a+bx)}{8b^3}-\frac{\sqrt{\pi}\, \text{Erfi}(a+bx)}{8b^3}-\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left(\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[(a + b*x)^2],x]
```

```
[Out] -((a*Cosh[(a + b*x)^2])/b^3) + ((a + b*x)*Cosh[(a + b*x)^2])/(2*b^3) - (Sqr
t[Pi]*Erf[a + b*x])/(8*b^3) - (a^2*Sqrt[Pi]*Erf[a + b*x])/(4*b^3) - (Sqrt[P
i]*Erfi[a + b*x])/(8*b^3) + (a^2*Sqrt[Pi]*Erfi[a + b*x])/(4*b^3)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5299

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5324

```
Int[((e_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Simp[(e^
(n - 1)*(e*x)^(m - n + 1)*Cosh[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1
))/(d*n), Int[(e*x)^(m - n)*Cosh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sinh\left((a+bx)^2\right)dx &= \frac{\text{Subst}\left(\int(-a+x)^2\sinh\left(x^2\right)dx,x,a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int\left(a^2\sinh\left(x^2\right)-2ax\sinh\left(x^2\right)+x^2\sinh\left(x^2\right)\right)dx,x,a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int x^2\sinh\left(x^2\right)dx,x,a+bx\right)}{b^3}-\frac{(2a)\,\text{Subst}\left(\int x\sinh\left(x^2\right)dx,x,a+bx\right)}{b^3}+\frac{a^2 \cdots}{\cdots}\\
&= \frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}-\frac{\text{Subst}\left(\int\cosh\left(x^2\right)dx,x,a+bx\right)}{2b^3}-\frac{a\,\text{Subst}\left(\int\sinh(x)\cdots\right.}{b^3}\\
&= -\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}-\frac{a^2\sqrt{\pi}\,\text{erf}(a+bx)}{4b^3}+\frac{a^2\sqrt{\pi}\,\text{erfi}(a\cdots}{4b^3}\\
&= -\frac{a\cosh\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\cosh\left((a+bx)^2\right)}{2b^3}-\frac{\sqrt{\pi}\,\text{erf}(a+bx)}{8b^3}-\frac{a^2\sqrt{\pi}\,\text{erf}(a+\cdots}{4b^3}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 63, normalized size = 0.56

$$
\frac{-\sqrt{\pi}\left(2a^2+1\right)\text{erf}(a+bx)+\sqrt{\pi}\left(2a^2-1\right)\text{erfi}(a+bx)-4(a-bx)\cosh\left((a+bx)^2\right)}{8b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[(a + b*x)^2],x]
```

```
[Out] (-4*(a - b*x)*Cosh[(a + b*x)^2] - (1 + 2*a^2)*Sqrt[Pi]*Erf[a + b*x] + (-1 +
 2*a^2)*Sqrt[Pi]*Erfi[a + b*x])/(8*b^3)
```

fricas [A] time = 0.53, size = 165, normalized size = 1.46

$$
-\frac{\left(\sqrt{\pi}\left(2\,a^2+1\right)\sqrt{b^2}\,\text{erf}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-\sqrt{\pi}\left(2\,a^2-1\right)\sqrt{b^2}\,\text{erfi}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)}-2\,b^2x+\cdots\right.}{8\,b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(pi)*(2*a^2 + 1)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2
+ 2*a*b*x + a^2) - sqrt(pi)*(2*a^2 - 1)*sqrt(b^2)*erfi(sqrt(b^2)*(b*x + a)/
b)*e^(b^2*x^2 + 2*a*b*x + a^2) - 2*b^2*x + 2*a*b - 2*(b^2*x - a*b)*e^(2*b^2
*x^2 + 4*a*b*x + 2*a^2))*e^(-b^2*x^2 - 2*a*b*x - a^2)/b^4
```

giac [C] time = 0.15, size = 137, normalized size = 1.21

$$-\frac{\frac{i\sqrt{\pi}\left(2a^2-1\right)\operatorname{erf}\left(ib\left(x+\frac{a}{b}\right)\right)}{b}-\frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(b^2x^2+2abx+a^2\right)}}{b}}{8b^2}+\frac{\frac{\sqrt{\pi}\left(2a^2+1\right)\operatorname{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b}+\frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(-b^2x^2-2abx-a^2\right)}}{b}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(pi)*(2*a^2 - 1)*erf(I*b*(x + a/b))/b - 2*(b*(x + a/b) - 2*a)*e
^(b^2*x^2 + 2*a*b*x + a^2)/b)/b^2 + 1/8*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a
/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b^2
```

maple [C] time = 0.07, size = 136, normalized size = 1.20

$$\frac{x\,\mathrm{e}^{-(bx+a)^2}}{4b^2}-\frac{a\,\mathrm{e}^{-(bx+a)^2}}{4b^3}-\frac{a^2\operatorname{erf}(bx+a)\sqrt{\pi}}{4b^3}-\frac{\operatorname{erf}(bx+a)\sqrt{\pi}}{8b^3}+\frac{x\,\mathrm{e}^{(bx+a)^2}}{4b^2}-\frac{a\,\mathrm{e}^{(bx+a)^2}}{4b^3}-\frac{ia^2\sqrt{\pi}\ \operatorname{erf}(ibx+ia)}{4b^3}+\frac{i\sqrt{\pi}\ \mathrm{e}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh((b*x+a)^2),x)
```

```
[Out] 1/4/b^2*x*exp(-(b*x+a)^2)-1/4*a/b^3*exp(-(b*x+a)^2)-1/4*a^2*erf(b*x+a)*Pi^(
1/2)/b^3-1/8*erf(b*x+a)*Pi^(1/2)/b^3+1/4/b^2*x*exp((b*x+a)^2)-1/4*a/b^3*exp
((b*x+a)^2)-1/4*I*a^2/b^3*Pi^(1/2)*erf(I*b*x+I*a)+1/8*I/b^3*Pi^(1/2)*erf(I*
b*x+I*a)
```

maxima [B] time = 0.74, size = 817, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sinh((b*x + a)^2) + 1/6*((sqrt(pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt(
(b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x +
a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2
)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (
b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^
4*b^5*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(9/2
)) - 6*(b^2*x + a*b)^3*a^2*b^5*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a
*b)^2)^(3/2)*(-b^2)^(9/2)) + 4*a^3*b^5*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(9/2
) - (b^2*x + a*b)^5*b^5*gamma(5/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^
(5/2)*(-b^2)^(9/2)) + 4*a*b^5*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(9/2))*b
/sqrt(-b^2) + a*(sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)
```

```
) - 1)/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*e^((b^2*x + a*b)^2/b^2)/b^3
 + gamma(2, -(b^2*x + a*b)^2/b^2)/b^3 - 3*(b^2*x + a*b)^3*a*gamma(3/2, -(b^
2*x + a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))/b - sqrt(pi)*(b^2*x +
 a*b)*a^4*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^5*sqrt(-(b^2*x + a*b)^2/
b^2)) + 4*a^3*e^((b^2*x + a*b)^2/b^2)/b^4 - 4*a*gamma(2, -(b^2*x + a*b)^2/b
^2)/b^4 + 6*(b^2*x + a*b)^3*a^2*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^7*(-(b^
2*x + a*b)^2/b^2)^(3/2)) + (b^2*x + a*b)^5*gamma(5/2, -(b^2*x + a*b)^2/b^2)
/(b^9*(-(b^2*x + a*b)^2/b^2)^(5/2)))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \sinh\left((a+b\,x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh((a + b*x)^2),x)
```

```
[Out] int(x^2*sinh((a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh\left(a^2 + 2abx + b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh((b*x+a)**2),x)
```

```
[Out] Integral(x**2*sinh(a**2 + 2*a*b*x + b**2*x**2), x)
```

3.89 $\int x \sinh\left((a+bx)^2\right) dx$

Optimal. Leaf size=54

$$\frac{\sqrt{\pi}\, a\,\text{erf}(a+bx)}{4b^2}-\frac{\sqrt{\pi}\, a\,\text{erfi}(a+bx)}{4b^2}+\frac{\cosh\left((a+bx)^2\right)}{2b^2}$$

```
[Out] 1/2*cosh((b*x+a)^2)/b^2+1/4*a*erf(b*x+a)*Pi^(1/2)/b^2-1/4*a*erfi(b*x+a)*Pi^
(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5364, 6742, 5298, 2204, 2205, 5320, 2638}

$$\frac{\sqrt{\pi}\, a\,\text{Erf}(a+bx)}{4b^2}-\frac{\sqrt{\pi}\, a\,\text{Erfi}(a+bx)}{4b^2}+\frac{\cosh\left((a+bx)^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[(a + b*x)^2],x]
```

```
[Out] Cosh[(a + b*x)^2]/(2*b^2) + (a*Sqrt[Pi]*Erf[a + b*x])/(4*b^2) - (a*Sqrt[Pi]
*Erfi[a + b*x])/(4*b^2)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
```

```
[n, 1]
```

Rule 5320

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sinh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x\sinh\left((a+bx)^2\right)\,dx &= \frac{\text{Subst}\left(\int(-a+x)\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int\left(-a\sinh\left(x^2\right)+x\sinh\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int x\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}-\frac{a\,\text{Subst}\left(\int\sinh\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int\sinh(x)\,dx,x,(a+bx)^2\right)}{2b^2}+\frac{a\,\text{Subst}\left(\int e^{-x^2}\,dx,x,a+bx\right)}{2b^2}-\frac{a\,\text{Subst}\left(\int e^{x^2}\,d\right.}{2b^2}\\
&= \frac{\cosh\left((a+bx)^2\right)}{2b^2}+\frac{a\sqrt{\pi}\,\text{erf}(a+bx)}{4b^2}-\frac{a\sqrt{\pi}\,\text{erfi}(a+bx)}{4b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.81

$$\frac{\cosh\left((a+bx)^2\right)}{2b^2}-\frac{\sqrt{\pi}\,a(\text{erfi}(a+bx)-\text{erf}(a+bx))}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[(a + b*x)^2],x]
```

```
[Out] Cosh[(a + b*x)^2]/(2*b^2) - (a*Sqrt[Pi]*(-Erf[a + b*x] + Erfi[a + b*x]))/(4
*b^2)
```

fricas [B] time = 0.55, size = 134, normalized size = 2.48

$$\frac{\left(\sqrt{\pi}\, a\sqrt{b^2}\ \mathrm{erf}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)} - \sqrt{\pi}\, a\sqrt{b^2}\ \mathrm{erfi}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)e^{\left(b^2x^2+2\,abx+a^2\right)} + be^{\left(2\,b^2x^2+4\,abx+2\,a^2\right)} + b\right)e^{\left(-b^2x\right.}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*a*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 + 2*a*b*x +
 a^2) - sqrt(pi)*a*sqrt(b^2)*erfi(sqrt(b^2)*(b*x + a)/b)*e^(b^2*x^2 + 2*a*b
*x + a^2) + b*e^(2*b^2*x^2 + 4*a*b*x + 2*a^2) + b)*e^(-b^2*x^2 - 2*a*b*x -
a^2)/b^3
```

giac [C] time = 0.15, size = 99, normalized size = 1.83

$$-\frac{-\frac{i\sqrt{\pi}\,a\,\mathrm{erf}\left(i\,b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(b^2x^2+2\,abx+a^2\right)}}{b}}{4\,b} - \frac{\frac{\sqrt{\pi}\,a\,\mathrm{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{b} - \frac{e^{\left(-b^2x^2-2\,abx-a^2\right)}}{b}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*(-I*sqrt(pi)*a*erf(I*b*(x + a/b))/b - e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b
 - 1/4*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b
```

maple [C] time = 0.04, size = 66, normalized size = 1.22

$$\frac{\mathrm{e}^{-(bx+a)^2}}{4b^2} + \frac{a\,\mathrm{erf}\,(bx+a)\sqrt{\pi}}{4b^2} + \frac{\mathrm{e}^{(bx+a)^2}}{4b^2} + \frac{ia\sqrt{\pi}\ \mathrm{erf}\,(ibx+ia)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh((b*x+a)^2),x)
```

```
[Out] 1/4/b^2*exp(-(b*x+a)^2)+1/4*a*erf(b*x+a)*Pi^(1/2)/b^2+1/4/b^2*exp((b*x+a)^2
)+1/4*I*a/b^2*Pi^(1/2)*erf(I*b*x+I*a)
```

maxima [B] time = 0.67, size = 649, normalized size = 12.02

$$\frac{1}{2}x^2\sinh\left((bx+a)^2\right)+\frac{1}{4}\left(\frac{\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{\left(b^2x+ab\right)^2}}{b}\right)-1\right)}{\sqrt{\left(b^2x+ab\right)^2}\left(-b^2\right)^{\frac{5}{2}}}-\frac{\left(b^2x+ab\right)^3b^3\Gamma\left(\frac{3}{2},\frac{\left(b^2x+ab\right)^2}{b^2}\right)}{\left(\left(b^2x+ab\right)^2\right)^{\frac{3}{2}}\left(-b^2\right)^{\frac{5}{2}}}+\frac{2ab^3e^{\left(-\frac{\left(b^2x+ab\right)^2}{b^2}\right)}}{\left(-b^2\right)^{\frac{5}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\sqrt{\pi}\left(b^2x\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh((b*x + a)^2) + 1/4*((sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt(
(b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*
b)^3*b^3*gamma(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5
/2)) + 2*a*b^3*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*a/sqrt(-b^2) + (sqrt(
pi)*(b^2*x + a*b)*a^3*b^4*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt((b^2*x +
 a*b)^2)*(-b^2)^(7/2)) - 3*(b^2*x + a*b)^3*a*b^4*gamma(3/2, (b^2*x + a*b)^2
/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(7/2)) + 3*a^2*b^4*e^(-(b^2*x + a*b)^
2/b^2)/(-b^2)^(7/2) + b^4*gamma(2, (b^2*x + a*b)^2/b^2)/(-b^2)^(7/2))*b/sqr
t(-b^2) - a*(sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) -
1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) - 2*a*e^((b^2*x + a*b)^2/b^2)/b^2 - (b^
2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-(b^2*x + a*b)^2/b^2)^(
3/2)))/b + sqrt(pi)*(b^2*x + a*b)*a^3*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)
/(b^4*sqrt(-(b^2*x + a*b)^2/b^2)) - 3*a^2*e^((b^2*x + a*b)^2/b^2)/b^3 + gam
ma(2, -(b^2*x + a*b)^2/b^2)/b^3 - 3*(b^2*x + a*b)^3*a*gamma(3/2, -(b^2*x +
a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^(3/2)))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\sinh\left((a+b\,x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh((a + b*x)^2),x)
```

```
[Out] int(x*sinh((a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sinh\left(a^2+2abx+b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh((b*x+a)**2),x)
```

```
[Out] Integral(x*sinh(a**2 + 2*a*b*x + b**2*x**2), x)
```

3.90 $\int \sinh\left((a+bx)^2\right)\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{\pi}\,\mathrm{erfi}(a+bx)}{4b}-\frac{\sqrt{\pi}\,\mathrm{erf}(a+bx)}{4b}$$

```
[Out] -1/4*erf(b*x+a)*Pi^(1/2)/b+1/4*erfi(b*x+a)*Pi^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5310, 5298, 2204, 2205}

$$\frac{\sqrt{\pi}\,\mathrm{Erfi}(a+bx)}{4b}-\frac{\sqrt{\pi}\,\mathrm{Erf}(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[(a + b*x)^2],x]
```

```
[Out] -(Sqrt[Pi]*Erf[a + b*x])/(4*b) + (Sqrt[Pi]*Erfi[a + b*x])/(4*b)
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5298

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sinh\left((a+bx)^2\right)\,dx &= \frac{\text{Subst}\left(\int \sinh\left(x^2\right)\,dx, x, a+bx\right)}{b}\\
&= -\frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, a+bx\right)}{2b} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, a+bx\right)}{2b}\\
&= -\frac{\sqrt{\pi}\,\text{erf}(a+bx)}{4b} + \frac{\sqrt{\pi}\,\text{erfi}(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.73

$$
\frac{\sqrt{\pi}\,(\text{erfi}(a+bx)-\text{erf}(a+bx))}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi]*(-Erf[a + b*x] + Erfi[a + b*x]))/(4*b)
```

fricas [A] time = 0.59, size = 55, normalized size = 1.49

$$
-\frac{\sqrt{\pi}\sqrt{b^2}\,\text{erf}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)-\sqrt{\pi}\sqrt{b^2}\,\text{erfi}\left(\frac{\sqrt{b^2}\,(bx+a)}{b}\right)}{4\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(b^2)*erf(sqrt(b^2)*(b*x + a)/b) - sqrt(pi)*sqrt(b^2)*er
fi(sqrt(b^2)*(b*x + a)/b))/b^2
```

giac [C] time = 0.13, size = 39, normalized size = 1.05

$$
-\frac{i\sqrt{\pi}\,\text{erf}\left(i\,b\left(x+\frac{a}{b}\right)\right)}{4\,b}+\frac{\sqrt{\pi}\,\text{erf}\left(-b\left(x+\frac{a}{b}\right)\right)}{4\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(pi)*erf(I*b*(x + a/b))/b + 1/4*sqrt(pi)*erf(-b*(x + a/b))/b
```

maple [C] time = 0.04, size = 36, normalized size = 0.97

$$-\frac{\operatorname{erf}(bx+a)\sqrt{\pi}}{4b}-\frac{i\sqrt{\pi}\ \operatorname{erf}(ibx+ia)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)^2),x)
```

```
[Out] -1/4*erf(b*x+a)*Pi^(1/2)/b-1/4*I*Pi^(1/2)/b*erf(I*b*x+I*a)
```

maxima [B] time = 0.58, size = 477, normalized size = 12.89

$$\frac{1}{2}\left(\frac{\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)ab^2\left(\operatorname{erf}\left(\frac{\sqrt{\left(b^2x+ab\right)^2}}{b}\right)-1\right)}{\sqrt{\left(b^2x+ab\right)^2}\left(-b^2\right)^{\frac{3}{2}}}+\frac{b^2e^{\left(-\frac{\left(b^2x+ab\right)^2}{b^2}\right)}}{\left(-b^2\right)^{\frac{3}{2}}}\right)a}{\sqrt{-b^2}}+\frac{\left(\frac{\sqrt{\pi}\left(b^2x+ab\right)a^2b^3\left(\operatorname{erf}\left(\frac{\sqrt{\left(b^2x+ab\right)^2}}{b}\right)-1\right)}{\sqrt{\left(b^2x+ab\right)^2}\left(-b^2\right)^{\frac{5}{2}}}-\frac{\left(b^2x+ab\right)^3b^3\Gamma\left(\frac{3}{2},\frac{\left(b^2x+ab\right)^2}{b^2}\right)}{\left(\left(b^2x+ab\right)^2\right)^{\frac{3}{2}}\left(-b^2\right)^{\frac{5}{2}}}+\right.}{\sqrt{-b^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((sqrt(pi)*(b^2*x + a*b)*a*b^2*(erf(sqrt((b^2*x + a*b)^2)/b) - 1)/(sqrt
((b^2*x + a*b)^2)*(-b^2)^(3/2)) + b^2*e^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(3/2)
)*a/sqrt(-b^2) + (sqrt(pi)*(b^2*x + a*b)*a^2*b^3*(erf(sqrt((b^2*x + a*b)^2)
/b) - 1)/(sqrt((b^2*x + a*b)^2)*(-b^2)^(5/2)) - (b^2*x + a*b)^3*b^3*gamma(3
/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^(3/2)*(-b^2)^(5/2)) + 2*a*b^3*e
^(-(b^2*x + a*b)^2/b^2)/(-b^2)^(5/2))*b/sqrt(-b^2) + a*(sqrt(pi)*(b^2*x + a
*b)*a*(erf(sqrt(-(b^2*x + a*b)^2/b^2)) - 1)/(b^2*sqrt(-(b^2*x + a*b)^2/b^2)
) - e^((b^2*x + a*b)^2/b^2)/b)/b - sqrt(pi)*(b^2*x + a*b)*a^2*(erf(sqrt(-(b
^2*x + a*b)^2/b^2)) - 1)/(b^3*sqrt(-(b^2*x + a*b)^2/b^2)) + 2*a*e^((b^2*x +
 a*b)^2/b^2)/b^2 + (b^2*x + a*b)^3*gamma(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-
(b^2*x + a*b)^2/b^2)^(3/2)))*b + x*sinh((b*x + a)^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sinh\left((a+b\,x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)^2),x)
```

```
[Out] int(sinh((a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sinh \left((a+bx)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)**2),x)
```

```
[Out] Integral(sinh((a + b*x)**2), x)
```

3.91 $\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$

Optimal. Leaf size=20

$$b\mathrm{Int}\left(\frac{\sinh\left((a+bx)^2\right)}{bx},x\right)$$

```
[Out] b*CannotIntegrate(sinh((b*x+a)^2)/b/x,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sinh[(a + b*x)^2]/x,x]
```

```
[Out] Defer[Subst][Defer[Int][Sinh[x^2]/(-a + x), x], x, a + b*x]
```

Rubi steps

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx = \mathrm{Subst}\left(\int \frac{\sinh\left(x^2\right)}{-a+x}\,dx,x,a+bx\right)$$

Mathematica [A] time = 9.98, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((a+bx)^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[(a + b*x)^2]/x,x]
```

```
[Out] Integrate[Sinh[(a + b*x)^2]/x, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sinh\left(b^2x^2+2\,abx+a^2\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(sinh(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(sinh((b*x + a)^2)/x, x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)^2)/x,x)
[Out] int(sinh((b*x+a)^2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(sinh((b*x + a)^2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sinh\left((a+b\,x)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)^2)/x,x)
[Out] int(sinh((a + b*x)^2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a^2 + 2abx + b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)**2)/x,x)
```

```
[Out] Integral(sinh(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

3.92 $\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\sinh\left((a+bx)^2\right)}{x^2},x\right)$$

[Out] Unintegrable(sinh((b*x+a)^2)/x^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[(a + b*x)^2]/x^2,x]

[Out] b*Defer[Subst][Defer[Int][Sinh[x^2]/(-a + x)^2, x], x, a + b*x]

Rubi steps

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx = b\,\text{Subst}\left(\int \frac{\sinh\left(x^2\right)}{(-a+x)^2} dx, x, a+bx\right)$$

Mathematica [A] time = 13.41, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((a+bx)^2\right)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[(a + b*x)^2]/x^2,x]

[Out] Integrate[Sinh[(a + b*x)^2]/x^2, x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh\left(b^2x^2 + 2\,abx + a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(sinh(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(sinh((b*x + a)^2)/x^2, x)
```

maple [A] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((b*x+a)^2)/x^2,x)
[Out] int(sinh((b*x+a)^2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(sinh((b*x + a)^2)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sinh\left((a+b\,x)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh((a + b*x)^2)/x^2,x)
[Out] int(sinh((a + b*x)^2)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a^2 + 2abx + b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh((b*x+a)**2)/x**2,x)
```

```
[Out] Integral(sinh(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

3.93 $\int x^2 \sinh\left(a + b\sqrt{c + dx}\right) dx$

Optimal. Leaf size=346

$$-\frac{240 \sinh\left(a + b\sqrt{c + dx}\right)}{b^6 d^3} + \frac{240\sqrt{c + dx} \cosh\left(a + b\sqrt{c + dx}\right)}{b^5 d^3} - \frac{120(c + dx) \sinh\left(a + b\sqrt{c + dx}\right)}{b^4 d^3} + \frac{24c \sinh\left(a\right.}{b^4}$$

```
[Out] 40*(d*x+c)^(3/2)*cosh(a+b*(d*x+c)^(1/2))/b^3/d^3-4*c*(d*x+c)^(3/2)*cosh(a+b
*(d*x+c)^(1/2))/b/d^3+2*(d*x+c)^(5/2)*cosh(a+b*(d*x+c)^(1/2))/b/d^3-240*sin
h(a+b*(d*x+c)^(1/2))/b^6/d^3+24*c*sinh(a+b*(d*x+c)^(1/2))/b^4/d^3-2*c^2*sin
h(a+b*(d*x+c)^(1/2))/b^2/d^3-120*(d*x+c)*sinh(a+b*(d*x+c)^(1/2))/b^4/d^3+12
*c*(d*x+c)*sinh(a+b*(d*x+c)^(1/2))/b^2/d^3-10*(d*x+c)^2*sinh(a+b*(d*x+c)^(1
/2))/b^2/d^3+240*cosh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3-24*c*cosh(a+
b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^3+2*c^2*cosh(a+b*(d*x+c)^(1/2))*(d*x+c
)^(1/2)/b/d^3
```

Rubi [A] time = 0.42, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5364, 5286, 3296, 2637}

$$-\frac{2c^2 \sinh\left(a + b\sqrt{c + dx}\right)}{b^2 d^3} - \frac{10(c + dx)^2 \sinh\left(a + b\sqrt{c + dx}\right)}{b^2 d^3} + \frac{12c(c + dx) \sinh\left(a + b\sqrt{c + dx}\right)}{b^2 d^3} - \frac{120(c + dx) \text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (240*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^5*d^3) - (24*c*Sqrt[c + d*
x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Cosh[a + b*S
qrt[c + d*x]])/(b*d^3) + (40*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b^
3*d^3) - (4*c*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c +
d*x)^(5/2)*Cosh[a + b*Sqrt[c + d*x]])/(b*d^3) - (240*Sinh[a + b*Sqrt[c + d*
x]])/(b^6*d^3) + (24*c*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (2*c^2*Sinh[a
 + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/
(b^4*d^3) + (12*c*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (10*(c +
 d*x)^2*Sinh[a + b*Sqrt[c + d*x]])/(b^2*d^3)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sinh\left(a+b\sqrt{c+dx}\right)dx = \frac{\text{Subst}\left(\int(-c+x)^2\sinh\left(a+b\sqrt{x}\right)dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int x\left(c-x^2\right)^2\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int\left(c^2x\sinh(a+bx)-2cx^3\sinh(a+bx)+x^5\sinh(a+bx)\right)dx, x, \sqrt{c+\ldots}\right.}{d^3}$$

$$= \frac{2\,\text{Subst}\left(\int x^5\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d^3} - \frac{(4c)\,\text{Subst}\left(\int x^3\sinh(a+bx)\,dx, x\ldots\right.}{d^3}$$

$$= \frac{2c^2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+\ldots}{\ldots}$$

$$= \frac{2c^2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} - \frac{4c(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{2(c+\ldots}{\ldots}$$

$$= -\frac{24c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c\ldots}{\ldots}$$

$$= -\frac{24c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^3} + \frac{40(c\ldots}{\ldots}$$

$$= \frac{240\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} - \frac{24c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c\ldots}}{\ldots}$$

$$= \frac{240\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^5d^3} - \frac{24c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3} + \frac{2c^2\sqrt{c\ldots}}{\ldots}$$

Mathematica [A] time = 1.30, size = 104, normalized size = 0.30

$$\frac{2b\sqrt{c+dx}\left(b^4d^2x^2+4b^2(2c+5dx)+120\right)\cosh\left(a+b\sqrt{c+dx}\right)-2\left(b^4dx(4c+5dx)+12b^2(4c+5dx)+120\right)s}{b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[c + d*x]*(120 + b^4*d^2*x^2 + 4*b^2*(2*c + 5*d*x))*Cosh[a + b*Sqr
t[c + d*x]] - 2*(120 + 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x))*Sinh[a
 + b*Sqrt[c + d*x]])/(b^6*d^3)
```

fricas [A] time = 0.54, size = 104, normalized size = 0.30

$$\frac{2\left(\left(b^5d^2x^2+20\,b^3dx+8\,b^3c+120\,b\right)\sqrt{dx+c}\,\cosh\left(\sqrt{dx+c}\,b+a\right)-\left(5\,b^4d^2x^2+48\,b^2c+4\left(b^4c+15\,b^2\right)dx+\right.\right.}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^5*d^2*x^2 + 20*b^3*d*x + 8*b^3*c + 120*b)*sqrt(d*x + c)*cosh(sqrt(d*x
 + c)*b + a) - (5*b^4*d^2*x^2 + 48*b^2*c + 4*(b^4*c + 15*b^2)*d*x + 120)*si
nh(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

giac [B] time = 0.16, size = 914, normalized size = 2.64

$$\frac{\left(\left(\sqrt{dx+c}\,b+a\right)b^4c^2-ab^4c^2-2\left(\sqrt{dx+c}\,b+a\right)^3b^2c+6\left(\sqrt{dx+c}\,b+a\right)^2ab^2c-6\left(\sqrt{dx+c}\,b+a\right)a^2b^2c+2\,a^3b^2c-b^4c^2+\left(\sqrt{dx+c}\,b+a\right)^5-5\left(\sqrt{dx+c}\,b+a\right)^4a+10\left(\sqrt{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^2
*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c
+ 2*a^3*b^2*c - b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)
^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*
(sqrt(d*x + c)*b + a)*a^4 - a^5 + 6*(sqrt(d*x + c)*b + a)^2*b^2*c - 12*(sqr
t(d*x + c)*b + a)*a*b^2*c + 6*a^2*b^2*c - 5*(sqrt(d*x + c)*b + a)^4 + 20*(s
qrt(d*x + c)*b + a)^3*a - 30*(sqrt(d*x + c)*b + a)^2*a^2 + 20*(sqrt(d*x + c
)*b + a)*a^3 - 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*c + 20*(sq
rt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*x + c)*b +
 a)*a^2 - 20*a^3 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x +
```

```
c)*b + a)*a - 60*a^2 + 120*sqrt(d*x + c)*b - 120)*e^(sqrt(d*x + c)*b + a)/(
b^5*d^2) + ((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b
+ a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*
a^2*b^2*c + 2*a^3*b^2*c + b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x +
 c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^
2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c
 + 12*(sqrt(d*x + c)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)
^4 - 20*(sqrt(d*x + c)*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sq
rt(d*x + c)*b + a)*a^3 + 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*
c + 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*
x + c)*b + a)*a^2 - 20*a^3 - 12*b^2*c + 60*(sqrt(d*x + c)*b + a)^2 - 120*(s
qrt(d*x + c)*b + a)*a + 60*a^2 + 120*sqrt(d*x + c)*b + 120)*e^(-sqrt(d*x +
c)*b - a)/(b^5*d^2))/(b*d)
```

maple [B] time = 0.02, size = 831, normalized size = 2.40

$$\frac{\frac{2\left(\left(a+b\sqrt{dx+c}\right)^5\cosh\left(a+b\sqrt{dx+c}\right)-5\left(a+b\sqrt{dx+c}\right)^4\sinh\left(a+b\sqrt{dx+c}\right)+20\left(a+b\sqrt{dx+c}\right)^3\cosh\left(a+b\sqrt{dx+c}\right)-60\sinh\left(a+b\sqrt{dx+c}\right)\left(a+b\sqrt{dx+c}\right)^2+120(}{b^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^3/b^2*(1/b^4*((a+b*(d*x+c)^(1/2))^5*cosh(a+b*(d*x+c)^(1/2))-5*(a+b*(d*x
+c)^(1/2))^4*sinh(a+b*(d*x+c)^(1/2))+20*(a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x
+c)^(1/2))-60*sinh(a+b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^2+120*(a+b*(d*x+c
)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-120*sinh(a+b*(d*x+c)^(1/2)))-5/b^4*a*((a+b
*(d*x+c)^(1/2))^4*cosh(a+b*(d*x+c)^(1/2))-4*(a+b*(d*x+c)^(1/2))^3*sinh(a+b*
(d*x+c)^(1/2))+12*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-24*sinh(a+b
*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))+24*cosh(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*
((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*sinh(a+b*(d*x+c)^(1/2))*(a
+b*(d*x+c)^(1/2))^2+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-6*sinh(a+
b*(d*x+c)^(1/2)))-10/b^4*a^3*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))
-2*sinh(a+b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/2)))-2
/b^2*c*((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*sinh(a+b*(d*x+c)^(1
/2))*(a+b*(d*x+c)^(1/2))^2+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-6*
sinh(a+b*(d*x+c)^(1/2)))+6/b^2*c*a*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^
(1/2))-2*sinh(a+b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/
2)))+5/b^4*a^4*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c
)^(1/2)))-6/b^2*a^2*c*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b
*(d*x+c)^(1/2)))-1/b^4*a^5*cosh(a+b*(d*x+c)^(1/2))+2/b^2*a^3*c*cosh(a+b*(d*
x+c)^(1/2))+c^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+
c)^(1/2)))-c^2*a*cosh(a+b*(d*x+c)^(1/2)))
```

maxima [A] time = 0.36, size = 486, normalized size = 1.40

$$\frac{2\,d^3x^3\sinh\left(\sqrt{dx+c}\,b+a\right)+\left(\frac{c^3e^{\left(\sqrt{dx+c}\,b+a\right)}}{b}-\frac{c^3e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b}-\frac{3\left((dx+c)b^2e^a-2\sqrt{dx+c}\,be^a+2\,e^a\right)c^2e^{\left(\sqrt{dx+c}\,b\right)}}{b^3}+\frac{3\left((dx+c)b^2+2\sqrt{dx\ldots}\right.}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*sinh(sqrt(d*x + c)*b + a) + (c^3*e^(sqrt(d*x + c)*b + a)/b -
 c^3*e^(-sqrt(d*x + c)*b - a)/b - 3*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c^2*e^(sqrt(d*x + c)*b)/b^3 + 3*((d*x + c)*b^2 + 2*sqrt(d*x +
c)*b + 2)*c^2*e^(-sqrt(d*x + c)*b - a)/b^3 + 3*((d*x + c)^2*b^4*e^a - 4*(d*
x + c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e
^a)*c*e^(sqrt(d*x + c)*b)/b^5 - 3*((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3
+ 12*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*c*e^(-sqrt(d*x + c)*b - a)/b^
5 - ((d*x + c)^3*b^6*e^a - 6*(d*x + c)^(5/2)*b^5*e^a + 30*(d*x + c)^2*b^4*e
^a - 120*(d*x + c)^(3/2)*b^3*e^a + 360*(d*x + c)*b^2*e^a - 720*sqrt(d*x + c
)*b*e^a + 720*e^a)*e^(sqrt(d*x + c)*b)/b^7 + ((d*x + c)^3*b^6 + 6*(d*x + c)
^(5/2)*b^5 + 30*(d*x + c)^2*b^4 + 120*(d*x + c)^(3/2)*b^3 + 360*(d*x + c)*b
^2 + 720*sqrt(d*x + c)*b + 720)*e^(-sqrt(d*x + c)*b - a)/b^7)*b)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\sinh\left(a+b\sqrt{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x^2*sinh(a + b*(c + d*x)^(1/2)), x)
```

sympy [A] time = 1.74, size = 269, normalized size = 0.78

$$\begin{cases} \frac{x^3\sinh(a)}{3} \\ \frac{x^3\sinh\left(a+b\sqrt{c}\right)}{3} \\ \frac{2x^2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{8cx\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{10x^2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{16c\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^3}+\frac{40x\sqrt{c+dx}\cosh\left(a+b\sqrt{\ldots}\right.}{b^3d^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*(d*x+c)**(1/2)),x)

[Out] Piecewise((x**3*sinh(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*sinh(a
+ b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/
(b*d) - 8*c*x*sinh(a + b*sqrt(c + d*x))/(b**2*d**2) - 10*x**2*sinh(a + b*sq
rt(c + d*x))/(b**2*d) + 16*c*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*
d**3) + 40*x*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*sin
h(a + b*sqrt(c + d*x))/(b**4*d**3) - 120*x*sinh(a + b*sqrt(c + d*x))/(b**4*
d**2) + 240*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**5*d**3) - 240*sinh(
a + b*sqrt(c + d*x))/(b**6*d**3), True))
```

3.94 $\int x \sinh\left(a + b\sqrt{c + dx}\right) dx$

Optimal. Leaf size=167

$$-\frac{12\sinh\left(a + b\sqrt{c + dx}\right)}{b^4 d^2} + \frac{12\sqrt{c + dx}\,\cosh\left(a + b\sqrt{c + dx}\right)}{b^3 d^2} - \frac{6(c + dx)\sinh\left(a + b\sqrt{c + dx}\right)}{b^2 d^2} + \frac{2c\sinh\left(a + b\sqrt{c \cdots\right.}}{b^2 d^2}$$

```
[Out] 2*(d*x+c)^(3/2)*cosh(a+b*(d*x+c)^(1/2))/b/d^2-12*sinh(a+b*(d*x+c)^(1/2))/b^
4/d^2+2*c*sinh(a+b*(d*x+c)^(1/2))/b^2/d^2-6*(d*x+c)*sinh(a+b*(d*x+c)^(1/2))
/b^2/d^2+12*cosh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*c*cosh(a+b*(d*x
+c)^(1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A] time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5364, 5286, 3296, 2637}

$$-\frac{6(c + dx)\sinh\left(a + b\sqrt{c + dx}\right)}{b^2 d^2} + \frac{2c\sinh\left(a + b\sqrt{c + dx}\right)}{b^2 d^2} - \frac{12\sinh\left(a + b\sqrt{c + dx}\right)}{b^4 d^2} + \frac{12\sqrt{c + dx}\,\cosh\left(a + b\sqrt{c \cdots\right.}}{b^3 d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (12*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]
*Cosh[a + b*Sqrt[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Cosh[a + b*Sqrt[c
+ d*x]])/(b*d^2) - (12*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (2*c*Sinh[a +
 b*Sqrt[c + d*x]])/(b^2*d^2) - (6*(c + d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^2
*d^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int x\sinh\left(a+b\sqrt{c+dx}\right)\,dx = \frac{\text{Subst}\left(\int(-c+x)\sinh\left(a+b\sqrt{x}\right)\,dx,x,c+dx\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x\left(-c+x^2\right)\sinh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int\left(-cx\sinh(a+bx)+x^3\sinh(a+bx)\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\,\text{Subst}\left(\int x^3\sinh(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2} - \frac{(2c)\,\text{Subst}\left(\int x\sinh(a+bx)\,dx,x,\right.}{d^2}$$
$$= -\frac{2c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{6\,\text{Subs}}{}$$
$$= -\frac{2c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2c\sinh}{}$$
$$= \frac{12\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)}{}$$
$$= \frac{12\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{2c\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)}{}$$

Mathematica [A] time = 0.21, size = 72, normalized size = 0.43

$$\frac{2b\left(b^2dx+6\right)\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)-2\left(b^2(2c+3dx)+6\right)\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[c + d*x]*(6 + b^2*d*x)*Cosh[a + b*Sqrt[c + d*x]] - 2*(6 + b^2*(2*
c + 3*d*x))*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

fricas [A] time = 0.64, size = 68, normalized size = 0.41

$$\frac{2\left(\left(b^{3}dx+6\,b\right)\sqrt{dx+c}\,\cosh\left(\sqrt{dx+c}\,b+a\right)-\left(3\,b^{2}dx+2\,b^{2}c+6\right)\sinh\left(\sqrt{dx+c}\,b+a\right)\right)}{b^{4}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^3*d*x + 6*b)*sqrt(d*x + c)*cosh(sqrt(d*x + c)*b + a) - (3*b^2*d*x + 2
*b^2*c + 6)*sinh(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

giac [B] time = 0.14, size = 299, normalized size = 1.79

$$-\frac{\frac{\left(\left(\sqrt{dx+c}\,b+a\right)b^{2}c-ab^{2}c-\left(\sqrt{dx+c}\,b+a\right)^{3}+3\left(\sqrt{dx+c}\,b+a\right)^{2}a-3\left(\sqrt{dx+c}\,b+a\right)a^{2}+a^{3}-b^{2}c+3\left(\sqrt{dx+c}\,b+a\right)^{2}-6\left(\sqrt{dx+c}\,b+a\right)a+3\,a^{2}-6\,\sqrt{dx+c}\,b+6\right)e^{\left(\sqrt{d\cdots}\right.}}{b^{3}d}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -(((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqr
t(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*b + a)*a^2 + a^3 - b^2*c + 3*(sqrt
(d*x + c)*b + a)^2 - 6*(sqrt(d*x + c)*b + a)*a + 3*a^2 - 6*sqrt(d*x + c)*b
+ 6)*e^(sqrt(d*x + c)*b + a)/(b^3*d) + ((sqrt(d*x + c)*b + a)*b^2*c - a*b^2
*c - (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x +
c)*b + a)*a^2 + a^3 + b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x + c)*
b + a)*a - 3*a^2 - 6*sqrt(d*x + c)*b - 6)*e^(-sqrt(d*x + c)*b - a)/(b^3*d))
/(b*d)
```

maple [B] time = 0.02, size = 303, normalized size = 1.81

$$\frac{\frac{2\left(\left(a+b\sqrt{dx+c}\right)^{3}\cosh\left(a+b\sqrt{dx+c}\right)-3\sinh\left(a+b\sqrt{dx+c}\right)\left(a+b\sqrt{dx+c}\right)^{2}+6\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\sqrt{dx+c}\right)-6\sinh\left(a+b\sqrt{dx+c}\right)\right)}{b^{2}}-\frac{6a\left(\left(a+b\sqrt{dx+c}\right.\right.\cdots}{\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^2/b^2*(1/b^2*((a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))-3*sinh(a+b*
(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^2+6*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)
^(1/2))-6*sinh(a+b*(d*x+c)^(1/2)))-3/b^2*a*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*
(d*x+c)^(1/2))-2*sinh(a+b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*
x+c)^(1/2)))+3/b^2*a^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+
```

```
b*(d*x+c)^(1/2)))-1/b^2*a^3*cosh(a+b*(d*x+c)^(1/2))-c*((a+b*(d*x+c)^(1/2))*
cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)))+c*a*cosh(a+b*(d*x+c)^(1/2)
))
```

maxima [A] time = 0.33, size = 293, normalized size = 1.75

$$\frac{2\,d^2x^2\sinh\left(\sqrt{dx+c}\,b+a\right)-\left(\frac{c^2e^{\left(\sqrt{dx+c}\,b+a\right)}}{b}-\frac{c^2e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b}-\frac{2\left((dx+c)b^2e^a-2\sqrt{dx+c}\,be^a+2\,e^a\right)ce^{\left(\sqrt{dx+c}\,b\right)}}{b^3}+\frac{2\left((dx+c)b^2+2\sqrt{dx+c}\right.}{b^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*sinh(sqrt(d*x + c)*b + a) - (c^2*e^(sqrt(d*x + c)*b + a)/b -
 c^2*e^(-sqrt(d*x + c)*b - a)/b - 2*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c*e^(sqrt(d*x + c)*b)/b^3 + 2*((d*x + c)*b^2 + 2*sqrt(d*x + c)
*b + 2)*c*e^(-sqrt(d*x + c)*b - a)/b^3 + ((d*x + c)^2*b^4*e^a - 4*(d*x + c)
^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e^a)*e^
(sqrt(d*x + c)*b)/b^5 - ((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3 + 12*(d*x
+ c)*b^2 + 24*sqrt(d*x + c)*b + 24)*e^(-sqrt(d*x + c)*b - a)/b^5)*b)/d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\sinh\left(a+b\sqrt{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x*sinh(a + b*(c + d*x)^(1/2)), x)
```

sympy [A] time = 0.61, size = 151, normalized size = 0.90

$$\begin{cases} \frac{x^2\sinh(a)}{2} & \text{for } b \\ \frac{x^2\sinh\left(a+b\sqrt{c}\right)}{2} & \text{for } d \\ \frac{2x\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{4c\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{6x\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{12\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{b^3d^2}-\frac{12\sinh\left(a+b\sqrt{c+dx}\right)}{b^4d^2} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**2*sinh(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*sinh(a
+ b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b*
d) - 4*c*sinh(a + b*sqrt(c + d*x))/(b**2*d**2) - 6*x*sinh(a + b*sqrt(c + d*
x))/(b**2*d) + 12*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*
sinh(a + b*sqrt(c + d*x))/(b**4*d**2), True))
```

3.95 $\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] -2*sinh(a+b*(d*x+c)^(1/2))/b^2/d+2*cosh(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/
d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5310, 5304, 3296, 2637}

$$\frac{2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd}-\frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]])/(b*d) - (2*Sinh[a + b*Sqrt[c +
d*x]])/(b^2*d)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Sinh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
```

```
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sinh\left(a+b\sqrt{c+dx}\right)\,dx &= \frac{\operatorname{Subst}\left(\int \sinh\left(a+b\sqrt{x}\right)\,dx, x, c+dx\right)}{d}\\
&= \frac{2\operatorname{Subst}\left(\int x\sinh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= \frac{2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\operatorname{Subst}\left(\int \cosh(a+bx)\,dx, x, \sqrt{c+dx}\right)}{bd}\\
&= \frac{2\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.93

$$\frac{2\left(b\sqrt{c+dx}\,\cosh\left(a+b\sqrt{c+dx}\right)-\sinh\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*(b*Sqrt[c + d*x]*Cosh[a + b*Sqrt[c + d*x]] - Sinh[a + b*Sqrt[c + d*x]]))
/(b^2*d)
```

fricas [A] time = 0.59, size = 44, normalized size = 0.81

$$\frac{2\left(\sqrt{dx+c}\,b\cosh\left(\sqrt{dx+c}\,b+a\right)-\sinh\left(\sqrt{dx+c}\,b+a\right)\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*b*cosh(sqrt(d*x + c)*b + a) - sinh(sqrt(d*x + c)*b + a))/(
b^2*d)
```

giac [A] time = 0.14, size = 64, normalized size = 1.19

$$\frac{\left(\sqrt{dx+c}\,b-1\right)e^{\left(\sqrt{dx+c}\,b+a\right)}}{b^2d}+\frac{\left(\sqrt{dx+c}\,b+1\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*b - 1)*e^(sqrt(d*x + c)*b + a)/(b^2*d) + (sqrt(d*x + c)*b +
1)*e^(-sqrt(d*x + c)*b - a)/(b^2*d)
```

maple [A] time = 0.01, size = 63, normalized size = 1.17

$$\frac{2\left(a+b\sqrt{dx+c}\right)\cosh\left(a+b\sqrt{dx+c}\right)-2\sinh\left(a+b\sqrt{dx+c}\right)-2a\cosh\left(a+b\sqrt{dx+c}\right)}{d\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d/b^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)
)-a*cosh(a+b*(d*x+c)^(1/2)))
```

maxima [B] time = 0.32, size = 111, normalized size = 2.06

$$-\frac{b\left(\frac{\left((dx+c)b^{2}e^{a}-2\sqrt{dx+c}\,be^{a}+2e^{a}\right)e^{\left(\sqrt{dx+c}\,b\right)}}{b^{3}}-\frac{\left((dx+c)b^{2}+2\sqrt{dx+c}\,b+2\right)e^{\left(-\sqrt{dx+c}\,b-a\right)}}{b^{3}}\right)-2\,(dx+c)\sinh\left(\sqrt{dx+c}\,b+a\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*((((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*e^a + 2*e^a)*e^(sqrt(d*x +
c)*b)/b^3 - ((d*x + c)*b^2 + 2*sqrt(d*x + c)*b + 2)*e^(-sqrt(d*x + c)*b - a
)/b^3) - 2*(d*x + c)*sinh(sqrt(d*x + c)*b + a))/d
```

mupad [B] time = 0.44, size = 43, normalized size = 0.80

$$-\frac{2\,\left(\sinh\left(a+b\,\sqrt{c+d\,x}\right)-b\cosh\left(a+b\,\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] -(2*(sinh(a + b*(c + d*x)^(1/2)) - b*cosh(a + b*(c + d*x)^(1/2))*(c + d*x)^
(1/2)))/(b^2*d)
```

sympy [A] time = 0.48, size = 65, normalized size = 1.20

$$\begin{cases} x\sinh(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x\sinh\left(a + b\sqrt{c}\right) & \text{for } d = 0 \\ \frac{2\sqrt{c+dx}\cosh\left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2\sinh\left(a+b\sqrt{c+dx}\right)}{b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sinh(a + b*sqrt
(c)), Eq(d, 0)), (2*sqrt(c + d*x)*cosh(a + b*sqrt(c + d*x))/(b*d) - 2*sinh(
a + b*sqrt(c + d*x))/(b**2*d), True))
```

3.96 $\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} dx$

Optimal. Leaf size=124

$$\sinh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\sinh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)-\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}\right.\right.$$

```
[Out] -cosh(a+b*c^(1/2))*Shi(b*(c^(1/2)-(d*x+c)^(1/2)))+cosh(a-b*c^(1/2))*Shi(b*(
c^(1/2)+(d*x+c)^(1/2)))+Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))+Ch
i(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5364, 5292, 3303, 3298, 3301}

$$\sinh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\sinh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)-\cosh\left(a+b\sqrt{c}\right)\text{Shi}\left(b\left(\sqrt{c}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])]*Sinh[a - b*Sqrt[c]] + CoshIntegra
l[b*(Sqrt[c] - Sqrt[c + d*x])]*Sinh[a + b*Sqrt[c]] - Cosh[a + b*Sqrt[c]]*Si
nhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Cosh[a - b*Sqrt[c]]*SinhIntegral[
b*(Sqrt[c] + Sqrt[c + d*x])]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt{x}\right)}{-c+x}\,dx, x, c+dx\right)$$

$$= 2\,\text{Subst}\left(\int \frac{x\sinh(a+bx)}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= 2\,\text{Subst}\left(\int\left(-\frac{\sinh(a+bx)}{2\left(\sqrt{c}-x\right)}+\frac{\sinh(a+bx)}{2\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c+dx}\right)$$

$$= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt{c}+x}\,dx, x, \sqrt{c+d\right.$$

$$= \cosh\left(a-b\sqrt{c}\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)+\cosh\left(a+b\sqrt{c}\right)\text{Su}$$

$$= \text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sinh\left(a-b\sqrt{c}\right)+\text{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)\sinh\left(a+b\sqrt{c}\right.$$

Mathematica [A] time = 0.99, size = 130, normalized size = 1.05

$$\frac{1}{2}e^{-a-b\sqrt{c}}\left(e^{2\left(a+b\sqrt{c}\right)}\text{Ei}\left(b\left(\sqrt{c+dx}-\sqrt{c}\right)\right)+e^{2a}\text{Ei}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)-\text{Ei}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)-e^{2b\sqrt{c}}\text{Ei}\left(-b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] (E^(-a - b*Sqrt[c])*(-ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])] + E^(2*(a
+ b*Sqrt[c]))*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - E^(2*b*Sqrt[c])
```

```
*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + E^(2*a)*ExpIntegralEi[b*(S
qrt[c] + Sqrt[c + d*x])]))/2
```

fricas [B] time = 0.58, size = 217, normalized size = 1.75

$$\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}\,b-\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right)+\frac{1}{2}\left(\mathrm{Ei}\left(\sqrt{dx+c}\,b+\sqrt{b^2c}\right)-\mathrm{Ei}\left(-\sqrt{dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b + sqrt(b^2*c))
)*cosh(a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - Ei(-sqrt
(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*
b - sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)
) - 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b - sqrt(b^2
*c)))*sinh(-a + sqrt(b^2*c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\sqrt{dx+c}\,b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+b\sqrt{dx+c}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/2))/x,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/2))/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\sqrt{dx+c}\,b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a+b\sqrt{c+d\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*(c + d*x)^(1/2))/x,x)
```

```
[Out] int(sinh(a + b*(c + d*x)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2))/x,x)
```

```
[Out] Integral(sinh(a + b*sqrt(c + d*x))/x, x)
```

3.97 $\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=182

$$\frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Sh}}{2\sqrt{}}$$

```
[Out] -sinh(a+b*(d*x+c)^(1/2))/x-1/2*b*d*Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*cosh(a-b*
c^(1/2))/c^(1/2)+1/2*b*d*Chi(b*(c^(1/2)-(d*x+c)^(1/2)))*cosh(a+b*c^(1/2))/c
^(1/2)-1/2*b*d*Shi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))/c^(1/2)-1/2
*b*d*Shi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))/c^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5364, 5288, 5281, 3303, 3298, 3301}

$$\frac{bd\cosh\left(a+b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\cosh\left(a-b\sqrt{c}\right)\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\sinh\left(a+b\sqrt{c}\right)\text{Sh}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (b*d*Cosh[a + b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt
[c]) - (b*d*Cosh[a - b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/
(2*Sqrt[c]) - Sinh[a + b*Sqrt[c + d*x]]/x - (b*d*Sinh[a + b*Sqrt[c]]*SinhIn
tegral[b*(Sqrt[c] - Sqrt[c + d*x])])/(2*Sqrt[c]) - (b*d*Sinh[a - b*Sqrt[c]]
*SinhIntegral[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = d\,\mathrm{Subst}\left(\int \frac{\sinh\left(a+b\sqrt{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\,\mathrm{Subst}\left(\int \frac{x\sinh(a+bx)}{\left(c-x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - (bd)\,\mathrm{Subst}\left(\int \frac{\cosh(a+bx)}{c-x^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - (bd)\,\mathrm{Subst}\left(\int \left(\frac{\cosh(a+bx)}{2\sqrt{c}\left(\sqrt{c}-x\right)} + \frac{\cosh(a+bx)}{2\sqrt{c}\left(\sqrt{c}+x\right)}\right) dx, x, \sqrt{}\right.$$

$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{(bd)\,\mathrm{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} - \frac{(bd)\,\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{\sinh\left(a+b\sqrt{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a-b\sqrt{c}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right.}{2\sqrt{c}}$$

$$= -\frac{bd\cosh\left(a-b\sqrt{c}\right)\mathrm{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}} + \frac{bd\cosh\left(a+b\sqrt{c}\right)\mathrm{Chi}\left(b\sqrt{c}-b\right.}{2\sqrt{c}}$$

Mathematica [A] time = 3.09, size = 199, normalized size = 1.09

$$\frac{e^{-a}\left(bdxe^{-b\sqrt{c}}\,\mathrm{Ei}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) - bdxe^{b\sqrt{c}}\,\mathrm{Ei}\left(-b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) + 2\sqrt{c}\,e^{-b\sqrt{c+dx}}\right) + e^{a}\left(bdxe^{b\sqrt{c}}\,\mathrm{Ei}\left(b\left(\sqrt{c}\right.\right.\right.}{4\sqrt{c}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (((2*Sqrt[c])/E^(b*Sqrt[c + d*x]) + (b*d*x*ExpIntegralEi[b*(Sqrt[c] - Sqrt[
c + d*x])])/E^(b*Sqrt[c]) - b*d*E^(b*Sqrt[c])*x*ExpIntegralEi[-(b*(Sqrt[c]
+ Sqrt[c + d*x]))])/E^a + E^a*(-2*Sqrt[c]*E^(b*Sqrt[c + d*x]) + b*d*E^(b*Sq
rt[c])*x*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - (b*d*x*ExpIntegralEi
[b*(Sqrt[c] + Sqrt[c + d*x])])/E^(b*Sqrt[c])))/(4*Sqrt[c]*x)
```

fricas [B] time = 0.49, size = 315, normalized size = 1.73

$$\frac{\left(\sqrt{b^2c}\,dx\mathrm{Ei}\left(\sqrt{dx+c}\,b-\sqrt{b^2c}\right) + \sqrt{b^2c}\,dx\mathrm{Ei}\left(-\sqrt{dx+c}\,b+\sqrt{b^2c}\right)\right)\cosh\left(a+\sqrt{b^2c}\right) - \left(\sqrt{b^2c}\,dx\mathrm{Ei}\left(\sqrt{dx+c}\,b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + sqrt(b^2*c)*d*x*E
i(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x
*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b -
sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) - 4*c*sinh(sqrt(d*x + c)*b + a) + (sqr
t(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d
*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) + (sqrt(b^2*c)*d*x*Ei(sqrt(
d*x + c)*b + sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*
c)))*sinh(-a + sqrt(b^2*c)))/(c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\sqrt{dx+c}\,b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+b\sqrt{dx+c}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/2))/x^2,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/2))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\sqrt{dx+c}\,b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sinh(sqrt(d*x + c)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(a + b\,\sqrt{c + d\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*(c + d*x)^(1/2))/x^2,x)
[Out] int(sinh(a + b*(c + d*x)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + b\sqrt{c + dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/2))/x**2,x)
[Out] Integral(sinh(a + b*sqrt(c + d*x))/x**2, x)
```

3.98 $\int x^2 \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=537

$$\frac{120960\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}-\frac{120960\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}+\frac{60480(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3}-$$

```
[Out] 120960*cosh(a+b*(d*x+c)^(1/3))/b^9/d^3+6*c^2*cosh(a+b*(d*x+c)^(1/3))/b^3/d^
3-720*c*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/3))/b^5/d^3+60480*(d*x+c)^(2/3)*c
osh(a+b*(d*x+c)^(1/3))/b^7/d^3+3*c^2*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/
b/d^3-120*c*(d*x+c)*cosh(a+b*(d*x+c)^(1/3))/b^3/d^3+5040*(d*x+c)^(4/3)*cosh
(a+b*(d*x+c)^(1/3))/b^5/d^3-6*c*(d*x+c)^(5/3)*cosh(a+b*(d*x+c)^(1/3))/b/d^3
+168*(d*x+c)^2*cosh(a+b*(d*x+c)^(1/3))/b^3/d^3+3*(d*x+c)^(8/3)*cosh(a+b*(d*
x+c)^(1/3))/b/d^3+720*c*sinh(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*(d*x+c)^(1/3
)*sinh(a+b*(d*x+c)^(1/3))/b^8/d^3-6*c^2*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3
))/b^2/d^3+360*c*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b^4/d^3-20160*(d*x+c
)*sinh(a+b*(d*x+c)^(1/3))/b^6/d^3+30*c*(d*x+c)^(4/3)*sinh(a+b*(d*x+c)^(1/3)
)/b^2/d^3-1008*(d*x+c)^(5/3)*sinh(a+b*(d*x+c)^(1/3))/b^4/d^3-24*(d*x+c)^(7/
3)*sinh(a+b*(d*x+c)^(1/3))/b^2/d^3
```

Rubi [A] time = 0.70, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5364, 1593, 5286, 3296, 2638, 2637}

$$-\frac{6c^2\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}+\frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}-\frac{24(c+dx)^{7/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{1008(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (120960*Cosh[a + b*(c + d*x)^(1/3)])/(b^9*d^3) + (6*c^2*Cosh[a + b*(c + d*x
)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)])/(
b^5*d^3) + (60480*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b^7*d^3) +
(3*c^2*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) - (120*c*(c + d
*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (5040*(c + d*x)^(4/3)*Cosh[a +
 b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^
(1/3)])/(b*d^3) + (168*(c + d*x)^2*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^3) +
 (3*(c + d*x)^(8/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^3) + (720*c*Sinh[a +
b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x
)^(1/3)])/(b^8*d^3) - (6*c^2*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(
b^2*d^3) + (360*c*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^3) -
(20160*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^3) + (30*c*(c + d*x)^(
4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*x)^(5/3)*Sinh[a
```

```
+ b*(c + d*x)^(1/3)])/(b^4*d^3) - (24*(c + d*x)^(7/3)*Sinh[a + b*(c + d*x)^
(1/3)])/(b^2*d^3)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int x^2 \sinh\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{\text{Subst}\left(\int(-c+x)^2 \sinh\left(a+b\sqrt[3]{x}\right) dx, x, c+dx\right)}{d^3}$$
$$= \frac{3\,\text{Subst}\left(\int\left(-cx+x^4\right)^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\left(-c+x^3\right)^2 \sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3}$$
$$= \frac{3\,\text{Subst}\left(\int\left(c^2x^2\sinh(a+bx)-2cx^5\sinh(a+bx)+x^8\sinh(a+bx)\right) dx, x, \sqrt[3]{c\right.}{d^3}$$
$$= \frac{3\,\text{Subst}\left(\int x^8\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} - \frac{(6c)\,\text{Subst}\left(\int x^5\sinh(a+bx)\,dx\right.}{d^3}$$
$$= \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(}{}$$
$$= \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(}{}$$
$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx}{}$$
$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx}{}$$
$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{3c^2(c+dx)^2}{}$$
$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{3c^2(c+dx)^2}{}$$
$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{60480(c+d}{}$$
$$= \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} + \frac{60480(c+d}{}$$
$$= \frac{120960\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3} + \frac{6c^2\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\cosh}{b^5d}$$

Mathematica [A] time = 2.90, size = 378, normalized size = 0.70

$$\frac{3\left((\sinh(a)+\cosh(a))\left(b^8d^2x^2(c+dx)^{2/3}-2b^7dx\sqrt[3]{c+dx}\,(3c+4dx)+2b^6\left(9c^2+36cdx+28d^2x^2\right)-24b^5(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*((40320 - 40320*b*(c + d*x)^(1/3) + 20160*b^2*(c + d*x)^(2/3) + b^8*d^2*
x^2*(c + d*x)^(2/3) - 2*b^7*d*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(c
+ d*x)^(1/3)*(6*c + 7*d*x) - 24*b^5*(c + d*x)^(2/3)*(9*c + 14*d*x) - 240*b^
3*(27*c + 28*d*x) + 2*b^6*(9*c^2 + 36*c*d*x + 28*d^2*x^2))*(Cosh[a] + Sinh[
a])*(Cosh[b*(c + d*x)^(1/3)] + Sinh[b*(c + d*x)^(1/3)]) + (40320 + 40320*b*
(c + d*x)^(1/3) + 20160*b^2*(c + d*x)^(2/3) + b^8*d^2*x^2*(c + d*x)^(2/3) +
 2*b^7*d*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 240*b^4*(c + d*x)^(1/3)*(6*c + 7
*d*x) + 24*b^5*(c + d*x)^(2/3)*(9*c + 14*d*x) + 240*b^3*(27*c + 28*d*x) + 2
*b^6*(9*c^2 + 36*c*d*x + 28*d^2*x^2))*(Cosh[a + b*(c + d*x)^(1/3)] - Sinh[a
 + b*(c + d*x)^(1/3)])))/(2*b^9*d^3)
```

fricas [A] time = 0.49, size = 180, normalized size = 0.34

$$\frac{3\left(\left(56\,b^6d^2x^2+72\,b^6cdx+18\,b^6c^2+\left(b^8d^2x^2+20160\,b^2\right)(dx+c)^{\frac{2}{3}}+240\left(7\,b^4dx+6\,b^4c\right)(dx+c)^{\frac{1}{3}}+40320\right)\cos\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*((56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 + (b^8*d^2*x^2 + 20160*b^2)*
(d*x + c)^(2/3) + 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) + 40320)*cosh((
d*x + c)^(1/3)*b + a) - 2*(3360*b^3*d*x + 3240*b^3*c + 12*(14*b^5*d*x + 9*b
^5*c)*(d*x + c)^(2/3) + (4*b^7*d^2*x^2 + 3*b^7*c*d*x + 20160*b)*(d*x + c)^(
1/3))*sinh((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

giac [B] time = 0.21, size = 2162, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*((((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c
^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*
b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(
1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*
c - 2*((d*x + c)^(1/3)*b + a)*b^6*c^2 + 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b +
a)^8 - 8*((d*x + c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 5
6*((d*x + c)^(1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d
*x + c)^(1/3)*b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c
)^(1/3)*b + a)*a^7 + a^8 + 10*((d*x + c)^(1/3)*b + a)^4*b^3*c - 40*((d*x +
c)^(1/3)*b + a)^3*a*b^3*c + 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c - 40*((d
```

*x + c)^(1/3)*b + a)*a^3*b^3*c + 10*a^4*b^3*c + 2*b^6*c^2 - 8*((d*x + c)^(1
/3)*b + a)^7 + 56*((d*x + c)^(1/3)*b + a)^6*a - 168*((d*x + c)^(1/3)*b + a)
^5*a^2 + 280*((d*x + c)^(1/3)*b + a)^4*a^3 - 280*((d*x + c)^(1/3)*b + a)^3*
a^4 + 168*((d*x + c)^(1/3)*b + a)^2*a^5 - 56*((d*x + c)^(1/3)*b + a)*a^6 +
8*a^7 - 40*((d*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*
a*b^3*c - 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x +
c)^(1/3)*b + a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)
*b + a)^4*a^2 - 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b
+ a)^2*a^4 - 336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 + 120*((d*x + c)^(1/
3)*b + a)^2*b^3*c - 240*((d*x + c)^(1/3)*b + a)*a*b^3*c + 120*a^2*b^3*c - 3
36*((d*x + c)^(1/3)*b + a)^5 + 1680*((d*x + c)^(1/3)*b + a)^4*a - 3360*((d*
x + c)^(1/3)*b + a)^3*a^2 + 3360*((d*x + c)^(1/3)*b + a)^2*a^3 - 1680*((d*x
+ c)^(1/3)*b + a)*a^4 + 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*
a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a
+ 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 +
1680*a^4 + 240*b^3*c - 6720*((d*x + c)^(1/3)*b + a)^3 + 20160*((d*x + c)^(
1/3)*b + a)^2*a - 20160*((d*x + c)^(1/3)*b + a)*a^2 + 6720*a^3 + 20160*((d*
x + c)^(1/3)*b + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 - 40320
*(d*x + c)^(1/3)*b + 40320)*e^((d*x + c)^(1/3)*b + a)/(b^8*d^2) + (((d*x +
c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c
^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3
c - 20((d*x + c)^(1/3)*b + a)^3*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*
a^3*b^3*c - 10*((d*x + c)^(1/3)*b + a)*a^4*b^3*c + 2*a^5*b^3*c + 2*((d*x +
c)^(1/3)*b + a)*b^6*c^2 - 2*a*b^6*c^2 + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x
+ c)^(1/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(
1/3)*b + a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*
b + a)^3*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)
a^7 + a^8 - 10((d*x + c)^(1/3)*b + a)^4*b^3*c + 40*((d*x + c)^(1/3)*b + a
)^3*a*b^3*c - 60*((d*x + c)^(1/3)*b + a)^2*a^2*b^3*c + 40*((d*x + c)^(1/3)*
b + a)*a^3*b^3*c - 10*a^4*b^3*c + 2*b^6*c^2 + 8*((d*x + c)^(1/3)*b + a)^7 -
56*((d*x + c)^(1/3)*b + a)^6*a + 168*((d*x + c)^(1/3)*b + a)^5*a^2 - 280*(
(d*x + c)^(1/3)*b + a)^4*a^3 + 280*((d*x + c)^(1/3)*b + a)^3*a^4 - 168*((d*
x + c)^(1/3)*b + a)^2*a^5 + 56*((d*x + c)^(1/3)*b + a)*a^6 - 8*a^7 - 40*((d
*x + c)^(1/3)*b + a)^3*b^3*c + 120*((d*x + c)^(1/3)*b + a)^2*a*b^3*c - 120*
((d*x + c)^(1/3)*b + a)*a^2*b^3*c + 40*a^3*b^3*c + 56*((d*x + c)^(1/3)*b +
a)^6 - 336*((d*x + c)^(1/3)*b + a)^5*a + 840*((d*x + c)^(1/3)*b + a)^4*a^2
- 1120*((d*x + c)^(1/3)*b + a)^3*a^3 + 840*((d*x + c)^(1/3)*b + a)^2*a^4 -
336*((d*x + c)^(1/3)*b + a)*a^5 + 56*a^6 - 120*((d*x + c)^(1/3)*b + a)^2*b^
3*c + 240*((d*x + c)^(1/3)*b + a)*a*b^3*c - 120*a^2*b^3*c + 336*((d*x + c)^
(1/3)*b + a)^5 - 1680*((d*x + c)^(1/3)*b + a)^4*a + 3360*((d*x + c)^(1/3)*b
+ a)^3*a^2 - 3360*((d*x + c)^(1/3)*b + a)^2*a^3 + 1680*((d*x + c)^(1/3)*b
+ a)*a^4 - 336*a^5 - 240*((d*x + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680
*((d*x + c)^(1/3)*b + a)^4 - 6720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x
+ c)^(1/3)*b + a)^2*a^2 - 6720*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 24
0*b^3*c + 6720*((d*x + c)^(1/3)*b + a)^3 - 20160*((d*x + c)^(1/3)*b + a)^2*

```
a + 20160*((d*x + c)^(1/3)*b + a)*a^2 - 6720*a^3 + 20160*((d*x + c)^(1/3)*b
 + a)^2 - 40320*((d*x + c)^(1/3)*b + a)*a + 20160*a^2 + 40320*(d*x + c)^(1/
3)*b + 40320)*e^(-(d*x + c)^(1/3)*b - a)/(b^8*d^2))/(b*d)
```

maple [B] time = 0.02, size = 1815, normalized size = 3.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^3/b^3*(1/b^6*a^8*cosh(a+b*(d*x+c)^(1/3))+1/b^6*((a+b*(d*x+c)^(1/3))^8*c
osh(a+b*(d*x+c)^(1/3))-8*(a+b*(d*x+c)^(1/3))^7*sinh(a+b*(d*x+c)^(1/3))+56*(
a+b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5*sinh
(a+b*(d*x+c)^(1/3))+1680*(a+b*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-6720
*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+20160*(a+b*(d*x+c)^(1/3))^2*
cosh(a+b*(d*x+c)^(1/3))-40320*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))+4
0320*cosh(a+b*(d*x+c)^(1/3)))+c^2*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(
1/3))-2*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3
)))+c^2*a^2*cosh(a+b*(d*x+c)^(1/3))+10/b^3*a*c*((a+b*(d*x+c)^(1/3))^4*cosh(
a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+12*(a+b*
(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d
*x+c)^(1/3))+24*cosh(a+b*(d*x+c)^(1/3)))-20/b^3*a^2*c*((a+b*(d*x+c)^(1/3))^
3*cosh(a+b*(d*x+c)^(1/3))-3*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))^2+6
*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+b*(d*x+c)^(1/3)))+20/
b^3*a^3*c*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-2*sinh(a+b*(d*x+c)
^(1/3))*(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3)))-10/b^3*a^4*c*((a+b*(
d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))-56/b^6*a^3*(
(a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*sinh(
a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-60*sinh
(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))^2+120*(a+b*(d*x+c)^(1/3))*cosh(a+b*
(d*x+c)^(1/3))-120*sinh(a+b*(d*x+c)^(1/3)))-2/b^3*c*((a+b*(d*x+c)^(1/3))^5*
cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+20*
(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-60*sinh(a+b*(d*x+c)^(1/3))*(a
+b*(d*x+c)^(1/3))^2+120*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-120*sin
h(a+b*(d*x+c)^(1/3)))-2*c^2*a*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-
sinh(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)
^(1/3))-2*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1
/3)))-8/b^6*a^7*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+
c)^(1/3)))-8/b^6*a*((a+b*(d*x+c)^(1/3))^7*cosh(a+b*(d*x+c)^(1/3))-7*(a+b*(d
*x+c)^(1/3))^6*sinh(a+b*(d*x+c)^(1/3))+42*(a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d
*x+c)^(1/3))-210*(a+b*(d*x+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+840*(a+b*(d*
x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-2520*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*
x+c)^(1/3))^2+5040*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-5040*sinh(a+
b*(d*x+c)^(1/3)))+28/b^6*a^2*((a+b*(d*x+c)^(1/3))^6*cosh(a+b*(d*x+c)^(1/3))
```

```
-6*(a+b*(d*x+c)^(1/3))^5*sinh(a+b*(d*x+c)^(1/3))+30*(a+b*(d*x+c)^(1/3))^4*c
osh(a+b*(d*x+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+36
0*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-720*sinh(a+b*(d*x+c)^(1/3))
*(a+b*(d*x+c)^(1/3))+720*cosh(a+b*(d*x+c)^(1/3)))-56/b^6*a^5*((a+b*(d*x+c)^
(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-3*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/
3))^2+6*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+b*(d*x+c)^(1/3
)))+2/b^3*a^5*c*cosh(a+b*(d*x+c)^(1/3))+70/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*c
osh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*(d*x+c)^(1/3))+12*(
a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*sinh(a+b*(d*x+c)^(1/3))*(a+
b*(d*x+c)^(1/3))+24*cosh(a+b*(d*x+c)^(1/3))))
```

maxima [A] time = 0.33, size = 642, normalized size = 1.20

$$2\,d^3x^3\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)+\left(\frac{c^3e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}-\frac{c^3e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{3\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)c^2e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*d^3*x^3*sinh((d*x + c)^(1/3)*b + a) + (c^3*e^((d*x + c)^(1/3)*b + a)
/b - c^3*e^(-(d*x + c)^(1/3)*b - a)/b - 3*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c^2*e^((d*x + c)^(1/3)*b)/
b^4 + 3*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c
^2*e^(-(d*x + c)^(1/3)*b - a)/b^4 + 3*((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5
/3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x
 + c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*c*e^((d*x + c)^(
1/3)*b)/b^7 - 3*((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/
3)*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*
b + 720)*c*e^(-(d*x + c)^(1/3)*b - a)/b^7 - ((d*x + c)^3*b^9*e^a - 9*(d*x +
 c)^(8/3)*b^8*e^a + 72*(d*x + c)^(7/3)*b^7*e^a - 504*(d*x + c)^2*b^6*e^a +
3024*(d*x + c)^(5/3)*b^5*e^a - 15120*(d*x + c)^(4/3)*b^4*e^a + 60480*(d*x +
 c)*b^3*e^a - 181440*(d*x + c)^(2/3)*b^2*e^a + 362880*(d*x + c)^(1/3)*b*e^a
 - 362880*e^a)*e^((d*x + c)^(1/3)*b)/b^10 + ((d*x + c)^3*b^9 + 9*(d*x + c)^
(8/3)*b^8 + 72*(d*x + c)^(7/3)*b^7 + 504*(d*x + c)^2*b^6 + 3024*(d*x + c)^(
5/3)*b^5 + 15120*(d*x + c)^(4/3)*b^4 + 60480*(d*x + c)*b^3 + 181440*(d*x +
c)^(2/3)*b^2 + 362880*(d*x + c)^(1/3)*b + 362880)*e^(-(d*x + c)^(1/3)*b - a
)/b^10)*b)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\sinh\left(a+b\,(c+d\,x)^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sinh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x^2*sinh(a + b*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sinh\left(a + b\sqrt[3]{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x**2*sinh(a + b*(c + d*x)**(1/3)), x)
```

3.99 $\int x \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=261

$$-\frac{360\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{360\sqrt[3]{c+dx}\,\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{60(c+dx}{}$$

```
[Out] -6*c*cosh(a+b*(d*x+c)^(1/3))/b^3/d^2+360*(d*x+c)^(1/3)*cosh(a+b*(d*x+c)^(1/
3))/b^5/d^2-3*c*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b/d^2+60*(d*x+c)*cosh
(a+b*(d*x+c)^(1/3))/b^3/d^2+3*(d*x+c)^(5/3)*cosh(a+b*(d*x+c)^(1/3))/b/d^2-3
60*sinh(a+b*(d*x+c)^(1/3))/b^6/d^2+6*c*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3)
)/b^2/d^2-180*(d*x+c)^(2/3)*sinh(a+b*(d*x+c)^(1/3))/b^4/d^2-15*(d*x+c)^(4/3
)*sinh(a+b*(d*x+c)^(1/3))/b^2/d^2
```

Rubi [A] time = 0.31, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5364, 5286, 3296, 2638, 2637}

$$-\frac{15(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{6c\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{36}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-6*c*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*Cosh[a
+ b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)
^(1/3)])/(b*d^2) + (60*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (
3*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^(1/3)])/(b*d^2) - (360*Sinh[a + b*(c
 + d*x)^(1/3)])/(b^6*d^2) + (6*c*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)
])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^4*d^2)
- (15*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2*d^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5286

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sinh[(c_.) + (d_.)*(x
_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int x\sinh\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{\text{Subst}\left(\int(-c+x)\sinh\left(a+b\sqrt[3]{x}\right)dx,x,c+dx\right)}{d^2}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\left(-c+x^3\right)\sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}$$
$$= \frac{3\,\text{Subst}\left(\int\left(-cx^2\sinh(a+bx)+x^5\sinh(a+bx)\right)dx,x,\sqrt[3]{c+dx}\right)}{d^2}$$
$$= \frac{3\,\text{Subst}\left(\int x^5\sinh(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2} - \frac{(3c)\,\text{Subst}\left(\int x^2\sinh(a+bx)\,dx,\right.}{d^2}$$
$$= -\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} - \frac{15\,\text{S}}{}$$
$$= -\frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{3(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{6c\sqrt[3]{}}{}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60(c+dx)\,\text{co}}{}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} - \frac{3c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} + \frac{60(c+dx)\,\text{co}}{}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}}{}$$
$$= -\frac{6c\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{360\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3}}{}$$

Mathematica [A] time = 0.35, size = 118, normalized size = 0.45

$$\frac{3b\left(b^4dx(c+dx)^{2/3}+2b^2(9c+10dx)+120\sqrt[3]{c+dx}\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)-3\left(b^4\sqrt[3]{c+dx}(3c+5dx)+60b^2(c\right.}{b^6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3) + 2*b^2*(9*c + 10*d*x))
*Cosh[a + b*(c + d*x)^(1/3)] - 3*(120 + 60*b^2*(c + d*x)^(2/3) + b^4*(c + d
*x)^(1/3)*(3*c + 5*d*x))*Sinh[a + b*(c + d*x)^(1/3)])/(b^6*d^2)
```

fricas [A] time = 0.52, size = 109, normalized size = 0.42

$$\frac{3\left(\left((dx+c)^{\frac{2}{3}}b^5dx+20\,b^3dx+18\,b^3c+120\,(dx+c)^{\frac{1}{3}}b\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(60\,(dx+c)^{\frac{2}{3}}b^2+\left(5\,b^4dx+3\,b\right.\right.\right.}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(((d*x + c)^(2/3)*b^5*d*x + 20*b^3*d*x + 18*b^3*c + 120*(d*x + c)^(1/3)*b
)*cosh((d*x + c)^(1/3)*b + a) - (60*(d*x + c)^(2/3)*b^2 + (5*b^4*d*x + 3*b^
4*c)*(d*x + c)^(1/3) + 120)*sinh((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

giac [B] time = 0.15, size = 706, normalized size = 2.70

$$-\frac{3\left(\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2 b^3c-2\left((dx+c)^{\frac{1}{3}}b+a\right)ab^3c+a^2b^3c-\left((dx+c)^{\frac{1}{3}}b+a\right)^5+5\left((dx+c)^{\frac{1}{3}}b+a\right)^4a-10\left((dx+c)^{\frac{1}{3}}b+a\right)^3a^2+10\left((dx+c)^{\frac{1}{3}}b+a\right)^2a^3-5\left((dx+c)^{\frac{1}{3}}b+\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3/2*((((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c
+ a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 1
0*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*
x + c)^(1/3)*b + a)*a^4 + a^5 - 2*((d*x + c)^(1/3)*b + a)*b^3*c + 2*a*b^3*c
 + 5*((d*x + c)^(1/3)*b + a)^4 - 20*((d*x + c)^(1/3)*b + a)^3*a + 30*((d*x
+ c)^(1/3)*b + a)^2*a^2 - 20*((d*x + c)^(1/3)*b + a)*a^3 + 5*a^4 + 2*b^3*c
- 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2*a - 60*((d*x
+ c)^(1/3)*b + a)*a^2 + 20*a^3 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*((d*x +
 c)^(1/3)*b + a)*a + 60*a^2 - 120*(d*x + c)^(1/3)*b + 120)*e^((d*x + c)^(1/
3)*b + a)/(b^5*d) + (((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b
 + a)*a*b^3*c + a^2*b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*
b + a)^4*a - 10*((d*x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^
2*a^3 - 5*((d*x + c)^(1/3)*b + a)*a^4 + a^5 + 2*((d*x + c)^(1/3)*b + a)*b^3
*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a)^4 + 20*((d*x + c)^(1/3)*b + a)^3
*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 20*((d*x + c)^(1/3)*b + a)*a^3 - 5*
a^4 + 2*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3 + 60*((d*x + c)^(1/3)*b + a)^2
*a - 60*((d*x + c)^(1/3)*b + a)*a^2 + 20*a^3 - 60*((d*x + c)^(1/3)*b + a)^2
 + 120*((d*x + c)^(1/3)*b + a)*a - 60*a^2 - 120*(d*x + c)^(1/3)*b - 120)*e^
(-(d*x + c)^(1/3)*b - a)/(b^5*d))/(b*d)
```

maple [B] time = 0.02, size = 659, normalized size = 2.52

$$\frac{\frac{3\left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^5\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-5\left(a+b(dx+c)^{\frac{1}{3}}\right)^4\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)+20\left(a+b(dx+c)^{\frac{1}{3}}\right)^3\cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)-60\sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)\left(a+b(dx+c)^{\frac{1}{3}}\cdots\right.\right.}{b^3}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^2/b^3*(1/b^3*((a+b*(d*x+c)^(1/3))^5*cosh(a+b*(d*x+c)^(1/3))-5*(a+b*(d*x
+c)^(1/3))^4*sinh(a+b*(d*x+c)^(1/3))+20*(a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x
+c)^(1/3))-60*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))^2+120*(a+b*(d*x+c
)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-120*sinh(a+b*(d*x+c)^(1/3)))-5/b^3*a*((a+b
*(d*x+c)^(1/3))^4*cosh(a+b*(d*x+c)^(1/3))-4*(a+b*(d*x+c)^(1/3))^3*sinh(a+b*
(d*x+c)^(1/3))+12*(a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-24*sinh(a+b
*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))+24*cosh(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*
((a+b*(d*x+c)^(1/3))^3*cosh(a+b*(d*x+c)^(1/3))-3*sinh(a+b*(d*x+c)^(1/3))*(a
+b*(d*x+c)^(1/3))^2+6*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-6*sinh(a+
b*(d*x+c)^(1/3)))-10/b^3*a^3*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))
-2*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3)))+5
/b^3*a^4*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3
)))-1/b^3*a^5*cosh(a+b*(d*x+c)^(1/3))-c*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*
x+c)^(1/3))-2*sinh(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c
)^(1/3)))+2*c*a*((a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+
c)^(1/3)))-c*a^2*cosh(a+b*(d*x+c)^(1/3)))
```

maxima [A] time = 0.32, size = 371, normalized size = 1.42

$$2\,d^2x^2\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left(\frac{c^2e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b}-\frac{c^2e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b}-\frac{2\left((dx+c)b^3e^a-3\,(dx+c)^{\frac{2}{3}}b^2e^a+6\,(dx+c)^{\frac{1}{3}}be^a-6\,e^a\right)ce^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d^2*x^2*sinh((d*x + c)^(1/3)*b + a) - (c^2*e^((d*x + c)^(1/3)*b + a)
/b - c^2*e^(-(d*x + c)^(1/3)*b - a)/b - 2*((d*x + c)*b^3*e^a - 3*(d*x + c)^
(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c*e^((d*x + c)^(1/3)*b)/b^
4 + 2*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c*e
^(-(d*x + c)^(1/3)*b - a)/b^4 + ((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5/3)*b^
5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x + c)^
(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*e^((d*x + c)^(1/3)*b)/
b^7 - ((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/3)*b^4 + 1
20*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*b + 720)*e
^(-(d*x + c)^(1/3)*b - a)/b^7)*b)/d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sinh \left(a + b\, (c + d\, x)^{1/3}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x*sinh(a + b*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sinh \left(a + b \sqrt[3]{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x*sinh(a + b*(c + d*x)**(1/3)), x)
```

3.100 $\int \sinh\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=85

$$\frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}-\frac{6\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[Out] 6*cosh(a+b*(d*x+c)^(1/3))/b^3/d+3*(d*x+c)^(2/3)*cosh(a+b*(d*x+c)^(1/3))/b/d
-6*(d*x+c)^(1/3)*sinh(a+b*(d*x+c)^(1/3))/b^2/d
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5310, 5304, 3296, 2638}

$$-\frac{6\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*Cosh[a + b*(c + d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Cosh[a + b*(c
+ d*x)^(1/3)])/(b*d) - (6*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^2
*d)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5304

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Sinh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5310

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\int \sinh\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{\text{Subst}\left(\int \sinh\left(a+b\sqrt[3]{x}\right)dx, x, c+dx\right)}{d}$$
$$= \frac{3\,\text{Subst}\left(\int x^2\sinh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\,\text{Subst}\left(\int x\cosh(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd}$$
$$= \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{6\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{3(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\sqrt[3]{c+dx}\,\sinh\left(a+\right.}{b^2d}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 0.76

$$\frac{3\left(b^2(c+dx)^{2/3}+2\right)\cosh\left(a+b\sqrt[3]{c+dx}\right)-6b\sqrt[3]{c+dx}\,\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*(2 + b^2*(c + d*x)^(2/3))*Cosh[a + b*(c + d*x)^(1/3)] - 6*b*(c + d*x)^(1
/3)*Sinh[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

fricas [A] time = 0.56, size = 58, normalized size = 0.68

$$-\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^2+2\right)\cosh\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*(2*(d*x + c)^(1/3)*b*sinh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2
+ 2)*cosh((d*x + c)^(1/3)*b + a))/(b^3*d)
```

giac [A] time = 0.13, size = 128, normalized size = 1.51

$$\frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^{2}-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^{2}-2\,(dx+c)^{\frac{1}{3}}b+2\right)e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{2\,b^{3}d}+\frac{3\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^{2}-2\left((dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2*(d*x
 + c)^(1/3)*b + 2)*e^((d*x + c)^(1/3)*b + a)/(b^3*d) + 3/2*(((d*x + c)^(1/3
)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 + 2*(d*x + c)^(1/3)*b + 2)*e
^(-(d*x + c)^(1/3)*b - a)/(b^3*d)
```

maple [A] time = 0.02, size = 133, normalized size = 1.56

$$\frac{3\left(a+b\,(dx+c)^{\frac{1}{3}}\right)^{2}\cosh\left(a+b\,(dx+c)^{\frac{1}{3}}\right)-6\sinh\left(a+b\,(dx+c)^{\frac{1}{3}}\right)\left(a+b\,(dx+c)^{\frac{1}{3}}\right)+6\cosh\left(a+b\,(dx+c)^{\frac{1}{3}}\right.}{d\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d/b^3*((a+b*(d*x+c)^(1/3))^2*cosh(a+b*(d*x+c)^(1/3))-2*sinh(a+b*(d*x+c)^(
1/3))*(a+b*(d*x+c)^(1/3))+2*cosh(a+b*(d*x+c)^(1/3))-2*a*((a+b*(d*x+c)^(1/3)
)*cosh(a+b*(d*x+c)^(1/3))-sinh(a+b*(d*x+c)^(1/3)))+a^2*cosh(a+b*(d*x+c)^(1/
3)))
```

maxima [A] time = 0.32, size = 137, normalized size = 1.61

$$-\frac{b\left(\frac{\left((dx+c)b^{3}e^{a}-3\,(dx+c)^{\frac{2}{3}}b^{2}e^{a}+6\,(dx+c)^{\frac{1}{3}}be^{a}-6\,e^{a}\right)e^{\left((dx+c)^{\frac{1}{3}}b\right)}}{b^{4}}-\frac{\left((dx+c)b^{3}+3\,(dx+c)^{\frac{2}{3}}b^{2}+6\,(dx+c)^{\frac{1}{3}}b+6\right)e^{\left(-(dx+c)^{\frac{1}{3}}b-a\right)}}{b^{4}}\right)-2\,(dx+c)\sinh}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(((d*x + c)*b^3*e^a - 3*(d*x + c)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)
*b*e^a - 6*e^a)*e^((d*x + c)^(1/3)*b)/b^4 - ((d*x + c)*b^3 + 3*(d*x + c)^(2
/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*e^(-(d*x + c)^(1/3)*b - a)/b^4) - 2*(d*x
 + c)*sinh((d*x + c)^(1/3)*b + a))/d
```

mupad [B] time = 0.47, size = 75, normalized size = 0.88

$$\frac{6\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{b^3\,d}+\frac{3\cosh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{2/3}}{b\,d}-\frac{6\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{1/3}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (6*cosh(a + b*(c + d*x)^(1/3)))/(b^3*d) + (3*cosh(a + b*(c + d*x)^(1/3))*(c
 + d*x)^(2/3))/(b*d) - (6*sinh(a + b*(c + d*x)^(1/3))*(c + d*x)^(1/3))/(b^2
*d)
```

sympy [A] time = 1.22, size = 94, normalized size = 1.11

$$\begin{cases} x\sinh\left(a\right) & \text{for}\ b=0 \wedge \left(b=0 \vee d=0\right) \\ x\sinh\left(a+b\sqrt[3]{c}\right) & \text{for}\ d=0 \\ \frac{3\left(c+dx\right)^{\frac{2}{3}}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{bd}-\frac{6\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{6\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sinh(a + b*c**(
1/3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*cosh(a + b*(c + d*x)**(1/3))/(b*d) -
6*(c + d*x)**(1/3)*sinh(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*cosh(a + b*(c
+ d*x)**(1/3))/(b**3*d), True))
```

3.101 $\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=232

$$\sinh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\sinh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)+\sinh\left(a+(-1)^{2/3}\right.$$

```
[Out] -cosh(a+b*c^(1/3))*Shi(b*(c^(1/3)-(d*x+c)^(1/3)))-cosh(a+(-1)^(2/3)*b*c^(1/
3))*Shi(b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))+cosh(a-(-1)^(1/3)*b*c^(1/3))*
Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))+Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*si
nh(a+b*c^(1/3))+Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)
*b*c^(1/3))+Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3)*b*
c^(1/3))
```

Rubi [A] time = 0.52, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5364, 5292, 3303, 3298, 3301}

$$\sinh\left(a+b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)+\sinh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Chi}\left(b\left(\sqrt[3]{-1}\,\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)+\sinh\left(a+(-1)^{2/3}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))]*Sinh[a + b*c^(1/3)] + CoshInteg
ral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]*Sinh[a - (-1)^(1/3)*b*c^(1/3)
] + CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))]*Sinh[a + (-1)
^(2/3)*b*c^(1/3)] - Cosh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)
^(1/3))] - Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3
) - (c + d*x)^(1/3))] + Cosh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)
^(1/3)*c^(1/3) + (c + d*x)^(1/3))]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx &= \text{Subst}\left(\int \frac{\sinh\left(a+b\sqrt[3]{x}\right)}{-c+x}\,dx, x, c+dx\right)\\
&= 3\,\text{Subst}\left(\int \frac{x^2\sinh(a+bx)}{-c+x^3}\,dx, x, \sqrt[3]{c+dx}\right)\\
&= 3\,\text{Subst}\left(\int \left(-\frac{\sinh(a+bx)}{3\left(\sqrt[3]{c}-x\right)}-\frac{\sinh(a+bx)}{3\left(-\sqrt[3]{-1}\sqrt[3]{c}-x\right)}-\frac{\sinh(a+bx)}{3\left((-1)^{2/3}\sqrt[3]{c}-x\right)}\right)dx, x, \sqrt[3]{c+\right.\\
&= -\text{Subst}\left(\int \frac{\sinh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)-\text{Subst}\left(\int \frac{\sinh(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+d\right.\\
&= \cosh\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{\sinh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)+\left(i\cosh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{\right.\right.\\
&= \text{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sinh\left(a+b\sqrt[3]{c}\right)+\text{Chi}\left(\sqrt[3]{-1}\,b\sqrt[3]{c}+b\sqrt[3]{c+dx}\right)\sinh\left(a-\sqrt[3]{\right.\right.
\end{aligned}$$

Mathematica [C] time = 0.07, size = 233, normalized size = 1.00

$$\frac{1}{2}\left(\text{RootSum}\left[c-\#1^3\&, \sinh(\#1b+a)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\cosh(\#1b+a)\text{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(\#\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] (-RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)
] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] - Cosh[a + b*#1]*
SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((c
+ d*x)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[
b*((c + d*x)^(1/3) - #1)] + CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a +
 b*#1] + Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b
*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

fricas [B] time = 0.67, size = 503, normalized size = 2.17

$$-\frac{1}{2}\operatorname{Ei}\left(-(dx+c)^{\frac{1}{3}}b-\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\right)\cosh\left(\frac{1}{2}\left(b^3c\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)-a\right)+\frac{1}{2}\operatorname{Ei}\left((dx+c)^{\frac{1}{3}}b-\frac{1}{2}\left(-b^3c\right)^{\frac{1}{3}}\left(\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] -1/2*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^
3*c)^(1/3)*(sqrt(-3) + 1) - a) + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1
/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - 1/2*Ei(-(
d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)
*(sqrt(-3) - 1) + a) + 1/2*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(
-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 1/2*Ei(-(d*x + c)^(
1/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) + 1/2*Ei((d*x + c)^(1/3)*b
+ (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) - 1/2*Ei(-(d*x + c)^(1/3)*b - 1
/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a)
 + 1/2*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(
-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + 1/2*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)
^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - 1/2*Ei(
(d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-b^3*c)^(1
/3)*(sqrt(-3) - 1) - a) + 1/2*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*sinh(a
 + (b^3*c)^(1/3)) - 1/2*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-
b^3*c)^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
```

```
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x, x)
```

maple [F]   time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + b\left(dx + c\right)^{\frac{1}{3}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/3))/x,x)
[Out] int(sinh(a+b*(d*x+c)^(1/3))/x,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(\left(dx + c\right)^{\frac{1}{3}}b + a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(a + b\left(c + d\,x\right)^{1/3}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*(c + d*x)^(1/3))/x,x)
[Out] int(sinh(a + b*(c + d*x)^(1/3))/x, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a + b\sqrt[3]{c + dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3))/x,x)
[Out] Integral(sinh(a + b*(c + d*x)**(1/3))/x, x)
```

3.102 $\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=329

$$\frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\mathrm{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}+\frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}$$

```
[Out] 1/3*b*d*Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*cosh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(
1/3)*b*d*Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*cosh(a-(-1)^(1/3)*b*c^(1
/3))/c^(2/3)+1/3*(-1)^(2/3)*b*d*Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*
cosh(a+(-1)^(2/3)*b*c^(1/3))/c^(2/3)-1/3*b*d*Shi(b*(c^(1/3)-(d*x+c)^(1/3)))
*sinh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(1/3)*b*d*Shi(b*((-1)^(1/3)*c^(1/3)+(d*
x+c)^(1/3)))*sinh(a-(-1)^(1/3)*b*c^(1/3))/c^(2/3)-1/3*(-1)^(2/3)*b*d*Shi(b*
((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3)*b*c^(1/3))/c^(2/3)-si
nh(a+b*(d*x+c)^(1/3))/x
```

Rubi [A] time = 0.72, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5364, 5288, 5281, 3303, 3298, 3301}

$$\frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\mathrm{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}+\frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\mathrm{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (b*d*Cosh[a + b*c^(1/3)]*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^
(2/3)) + ((-1)^(2/3)*b*d*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-(b*((
-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Cosh[
a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1
/3))])/(3*c^(2/3)) - Sinh[a + b*(c + d*x)^(1/3)]/x - (b*d*Sinh[a + b*c^(1/3
)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(2/3)*b
*d*Sinh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (c +
 d*x)^(1/3))])/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Sinh[a - (-1)^(1/3)*b*c^(1/3)]
*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))])/(3*c^(2/3))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5364

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
 0])^m*(a + b*Sinh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx = d\,\mathrm{Subst}\left(\int \frac{\sinh\left(a+b\sqrt[3]{x}\right)}{(-c+x)^2}\,dx, x, c+dx\right)$$
$$= (3d)\,\mathrm{Subst}\left(\int \frac{x^2\sinh(a+bx)}{\left(c-x^3\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)$$
$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\,\mathrm{Subst}\left(\int \frac{\cosh(a+bx)}{c-x^3}\,dx, x, \sqrt[3]{c+dx}\right)$$
$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - (bd)\,\mathrm{Subst}\left(\int\left(\frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-x\right)} + \frac{\cosh(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}+\sqrt[3]{-1}\,x\right)} + \right.\right.$$
$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{(bd)\,\mathrm{Subst}\left(\int \frac{\cosh(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{(bd)\,\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{\left(bd\cosh\left(a+b\sqrt[3]{c}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+d}\right.}{3c^{2/3}}$$
$$= \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\mathrm{Chi}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}} - \frac{\sqrt[3]{-1}\,bd\cosh\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\mathrm{Chi}\left(\right.}{3c^{2/3}}$$

Mathematica [C] time = 1.85, size = 210, normalized size = 0.64

$$\frac{e^{-a}\left(bdx\mathrm{RootSum}\left[c-\#1^3\&, \frac{-\sinh(\#1b)\mathrm{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\cosh(\#1b)\mathrm{Chi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\sinh(\#1b)\mathrm{Shi}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)-\cosh(\#1b)\mathrm{S}}{\#1^2}\right.\right.}{6x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] (b*d*x*RootSum[c - #1^3 & , (E^(a + b*#1)*ExpIntegralEi[b*((c + d*x)^(1/3)
- #1)])/#1^2 & ] + (3/E^(b*(c + d*x)^(1/3)) - 3*E^(2*a + b*(c + d*x)^(1/3))
 + b*d*x*RootSum[c - #1^3 & , (Cosh[b*#1]*CoshIntegral[b*((c + d*x)^(1/3) -
 #1)] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[b*#1] - Cosh[b*#1]*Sinh
Integral[b*((c + d*x)^(1/3) - #1)] + Sinh[b*#1]*SinhIntegral[b*((c + d*x)^(
1/3) - #1)])/#1^2 & ])/E^a)/(6*x)
```

fricas [B] time = 0.64, size = 704, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a + (
b^3*c)^(1/3)) - 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))
*cosh(-a + (-b^3*c)^(1/3)) - 2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b
^3*c)^(1/3))*sinh(a + (b^3*c)^(1/3)) + 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1
/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b^3*c)^(1/3)) - (b^3*c)^(1/3)*(sqrt(-3)
*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(
1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)
*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c
)^(1/3)*(sqrt(-3) + 1) + a) + (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x +
 c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)*(sqr
t(-3) - 1) + a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b
+ 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1)
 - a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3
*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) + (-b^
3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(
sqrt(-3) + 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - (b^3*c)^(1/3)*
(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) + (-b^3*c)^(1/3)*(sqrt(-3)*d
*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/
2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - 12*c*sinh((d*x + c)^(1/3)*b + a))/(c
*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x^2, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+b\,(dx+c)^{\frac{1}{3}}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a+b*(d*x+c)^(1/3))/x^2,x)
```

```
[Out] int(sinh(a+b*(d*x+c)^(1/3))/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
[Out] integrate(sinh((d*x + c)^(1/3)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*(c + d*x)^(1/3))/x^2,x)
[Out] int(sinh(a + b*(c + d*x)^(1/3))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(a+b*(d*x+c)**(1/3))/x**2,x)
[Out] Integral(sinh(a + b*(c + d*x)**(1/3))/x**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```